

Cubics

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

In general: $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$(a+b+c\dots)^3 = a^3 + b^3 + c^3 + \dots + 3a^2b + 3a^2c + 3ab^2 + \dots + 6abc$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

e.g. (i) $(2a-5)^3 = (2a)^3 - 3(2a)^2 5 + 3(2a)5^2 - 5^3$
 $= \underline{8a^3 - 60a^2 + 150a - 125}$

(ii) $x^3 + 8 = x^3 + 2^3$
 $= \underline{(x+2)(x^2 - 2x + 4)}$

(iii) $y^3 - 27a^3 = y^3 - (3a)^3$
 $= \underline{(y-3a)(y^2 + 3ay + 9a^2)}$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + a^2b^{n-3} + ab^{n-2} + b^{n-1})$$

$$(iv) \ x^5 - y^5 = (x - y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$$

$$(v) \ a^7 + 1 = a^7 - (-1)^7 \\ = (a + 1)(a^6 - a^5 + a^4 - a^3 + a^2 - a + 1)$$

Old Cambridge: Exercise 1E; 3dh, 4fjk, 5bh, 6bd, 7bc, 8bdik, 9bdf, 12*, 13*, 14*, 15*, 16*