

# *Equations of the form $a\sin x + b\cos x = c$*

## Using the $t$ results

eg (i)  $3\cos \theta + 4\sin \theta = 2$        $0 \leq \theta \leq 360^\circ$

let  $t = \tan \frac{\theta}{2}$        $3\left(\frac{1-t^2}{1+t^2}\right) + 4\left(\frac{2t}{1+t^2}\right) = 2$        $0 \leq \frac{\theta}{2} \leq 180^\circ$

$$3 - 3t^2 + 8t = 2 + 2t^2$$

$$5t^2 - 8t - 1 = 0$$

$$t = \frac{8 \pm \sqrt{84}}{10}$$

$$\tan \frac{\theta}{2} = \frac{4 - \sqrt{21}}{5}$$

or

$$Q2 \quad \tan \alpha = \frac{\sqrt{21} - 4}{5}$$

$$\alpha = 6^\circ 39'$$

$$\frac{\theta}{2} = 173^\circ 21'$$

$$\theta = 346^\circ 42'$$

$$\tan \frac{\theta}{2} = \frac{4 + \sqrt{21}}{5}$$

$$Q1 \quad \tan \alpha = \frac{4 + \sqrt{21}}{5}$$

$$\alpha = 59^\circ 47'$$

$$\frac{\theta}{2} = 59^\circ 47'$$

$$\theta = 119^\circ 33'$$

*Test :  $\theta = 180^\circ$*

$$3\cos 180^\circ + 4\sin 180^\circ \\ = -3 \neq 2$$

$$\therefore \theta = 119^\circ 33', 346^\circ 42'$$

**Exercise 11C;**  
**1, 3, 4, 6bc, 7, 8, 10, 11**