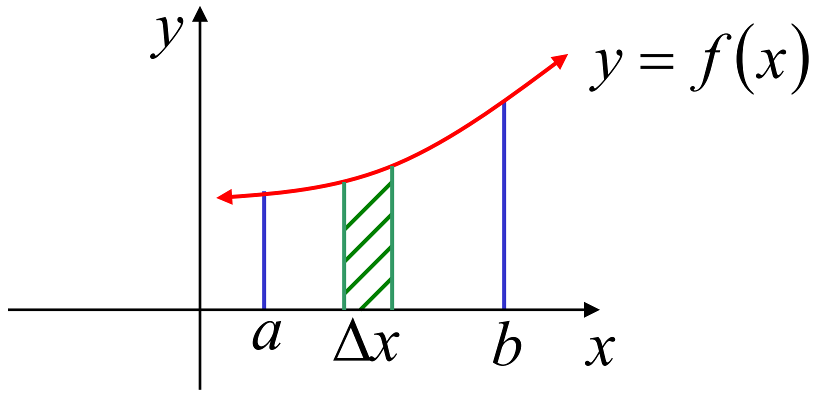
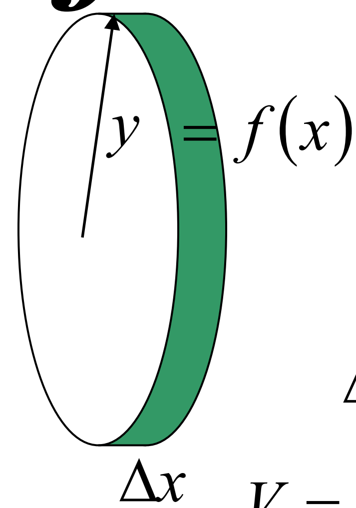
Volumes of Solids



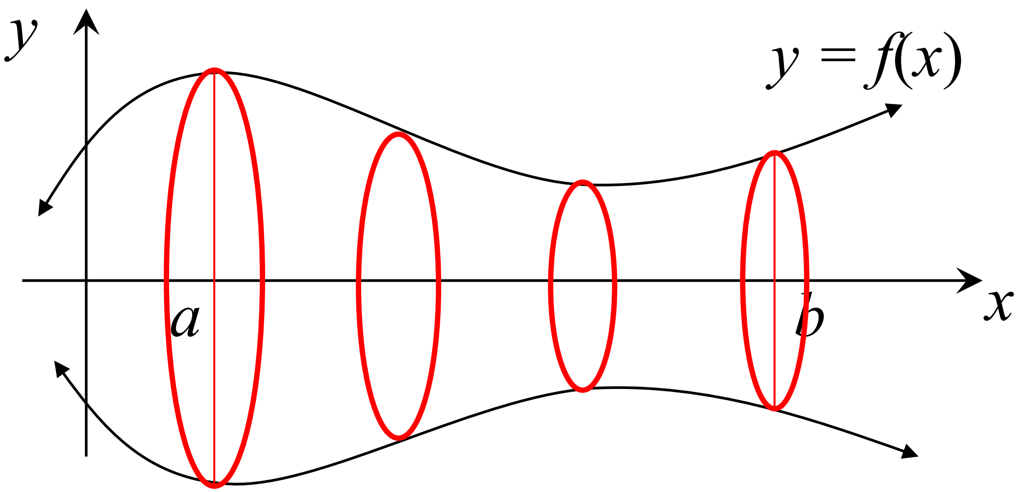


$$A(x) = \pi[f(x)]^2$$

$$\Delta V = \pi[f(x)]^2 \cdot \Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=a}^{b} \pi[f(x)]^2 \cdot \Delta x$$

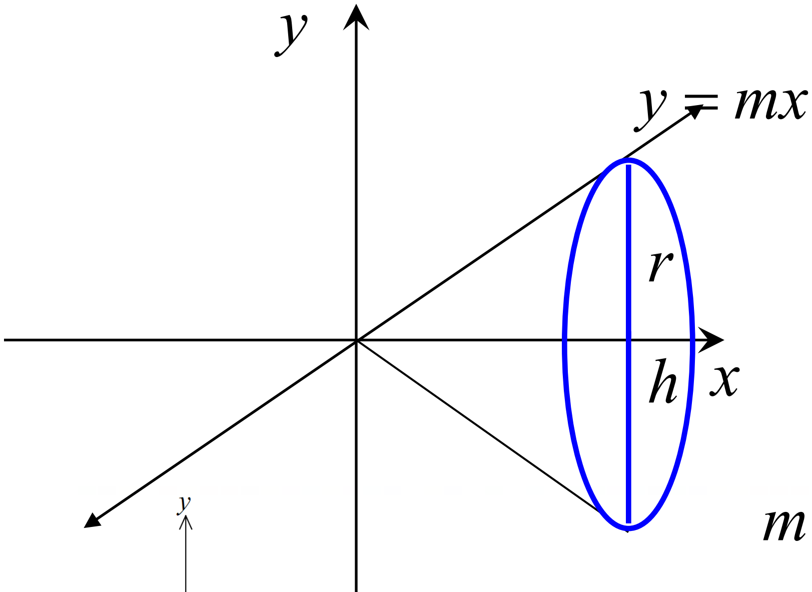
$$= \pi \int_a^b [f(x)]^2 \, dx$$



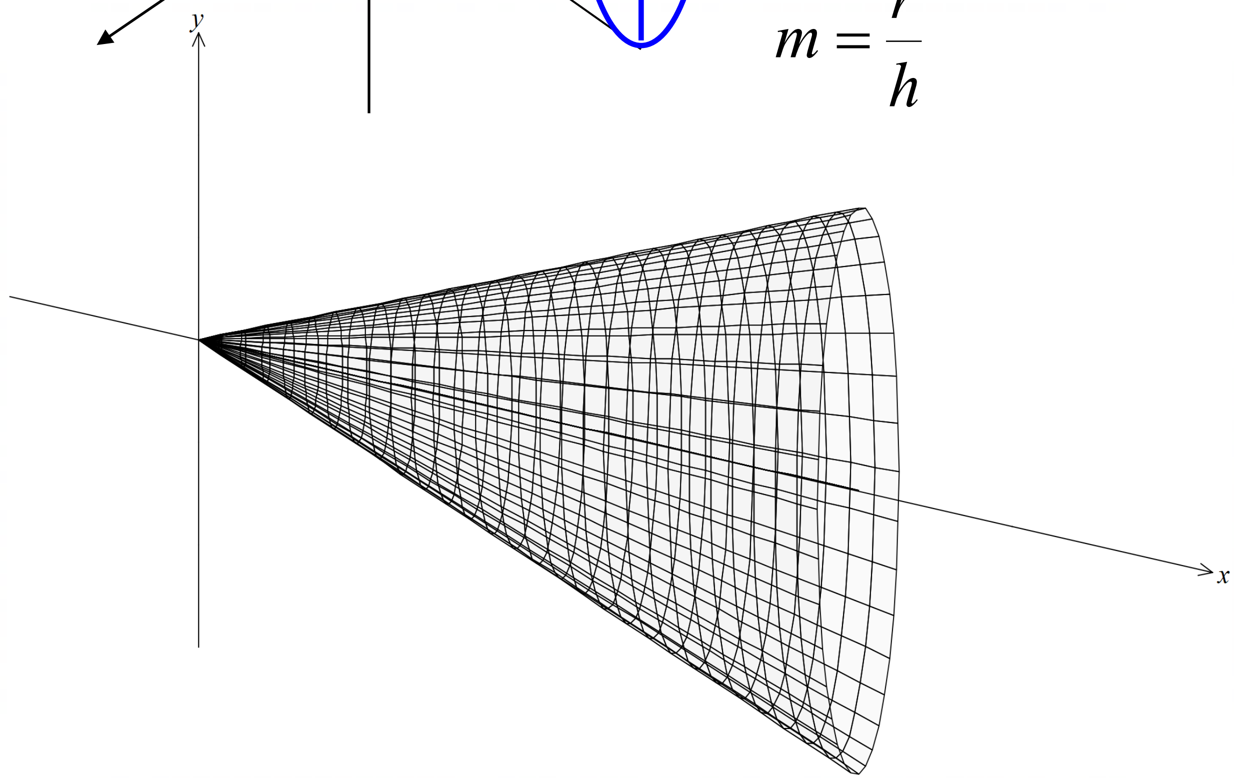
Volume of a solid of revolution about; (i) x axis : $V = \pi \int_a^b y^2 \, dx$

(ii) y axis : $V = \pi \int_c^d x^2 \, dy$

e.g. (i) <u>cone</u>



$$m = \frac{r}{h}$$



$$V = \pi \int y^2 dx$$

$$= \pi \int_0^h m^2 x^2 dx$$

$$= \pi m^2 \left[\frac{1}{3} x^3 \right]_0^h$$

$$= \frac{1}{3} \pi m^2 \left(h^3 - 0 \right)$$

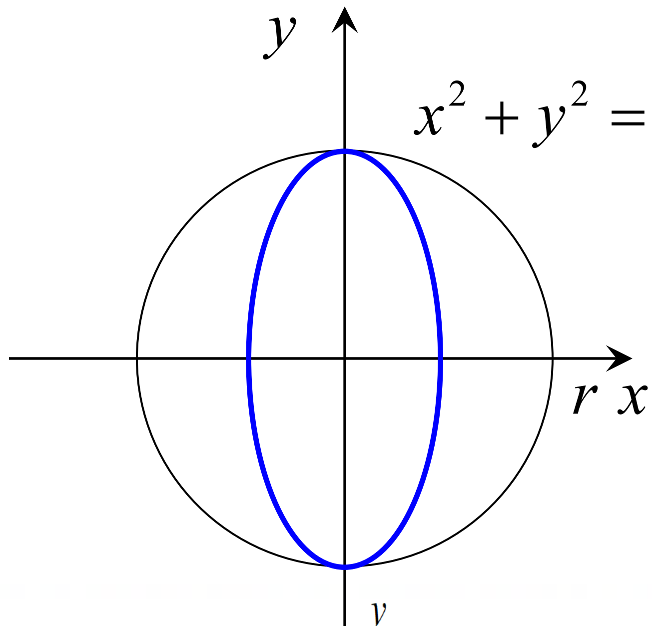
$$= \frac{1}{3} \pi m^2 h^3$$

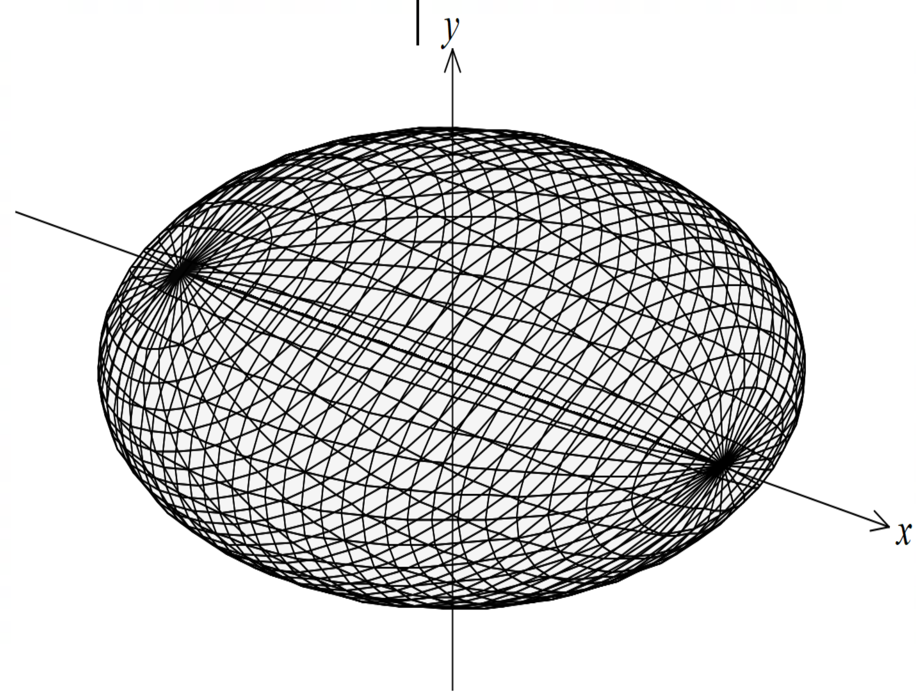
$$= \frac{1}{3} \pi \left(\frac{r}{h} \right)^2 h^3$$

$$= \frac{\pi r^2 h^3}{3h^2}$$

$$= \frac{\pi r^2 h}{3}$$

(*ii*) <u>sphere</u>





$$V = \pi \int y^2 dx$$

$$= 2\pi \int_0^r \left(r^2 - x^2\right) dx$$

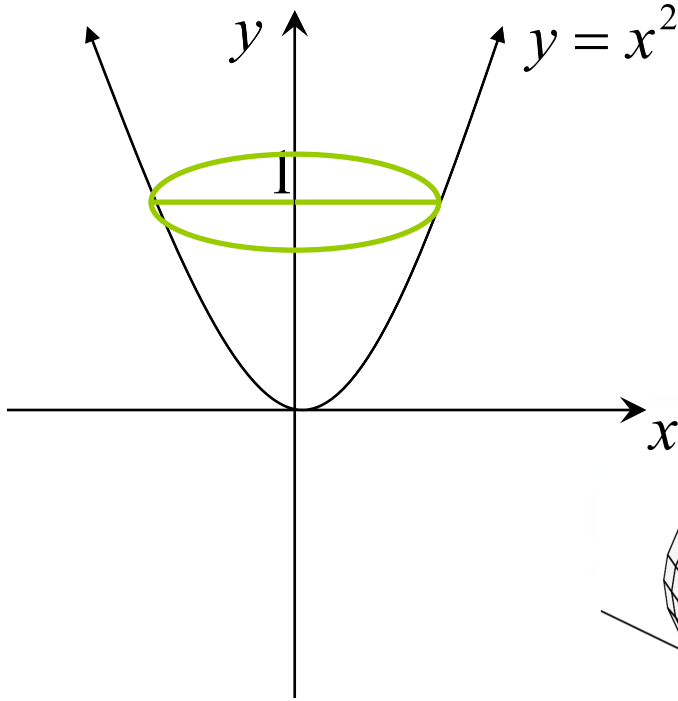
$$= 2\pi \left[r^2 x - \frac{1}{3} x^3 \right]_0^r$$

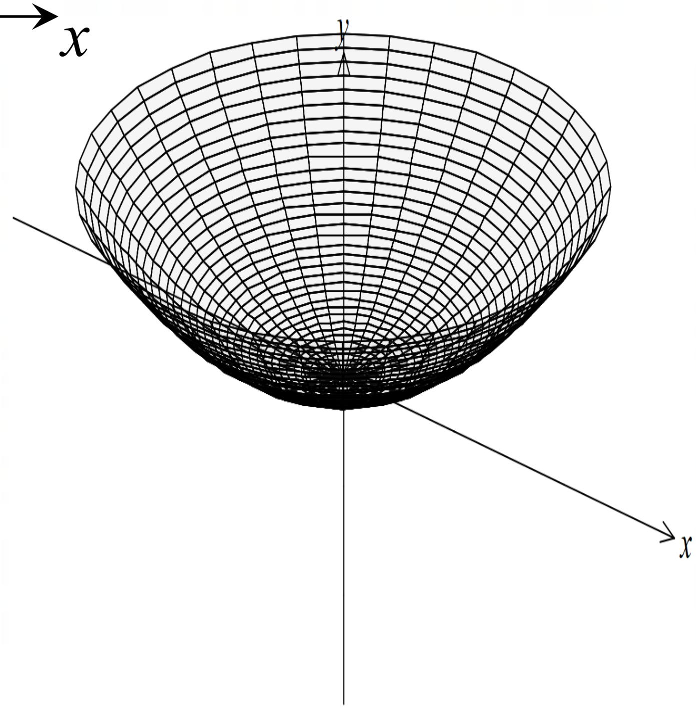
$$= 2\pi \left\{ \left(r^2 (r) - \frac{1}{3} r^3 \right) - 0 \right\}$$

$$= 2\pi \left(\frac{2}{3} r^3 \right)$$

$$= \frac{4}{3} \pi r^3$$

(iii) Find the volume of the solid generated when $y = x^2$ is revolved around the y axis between $y = 0$ and $y = 1$.





$$V = \pi \int x^2 dy$$

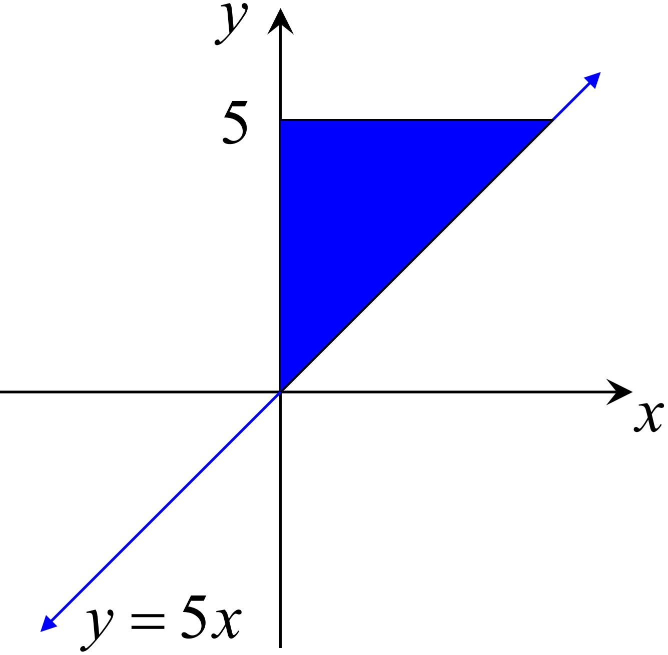
$$= \pi \int_0^1 y\,dy$$

$$= \frac{\pi}{2}\left[y^2\right]_0^1$$

$$= \frac{\pi}{2}\left(1^2 - 0\right)$$

$$= \frac{\pi}{2} \text{ units}^3$$

(*iv*) Find the volume of the solid when the shaded region is rotated about the x axis



$$V = \pi \int y^2 dx$$

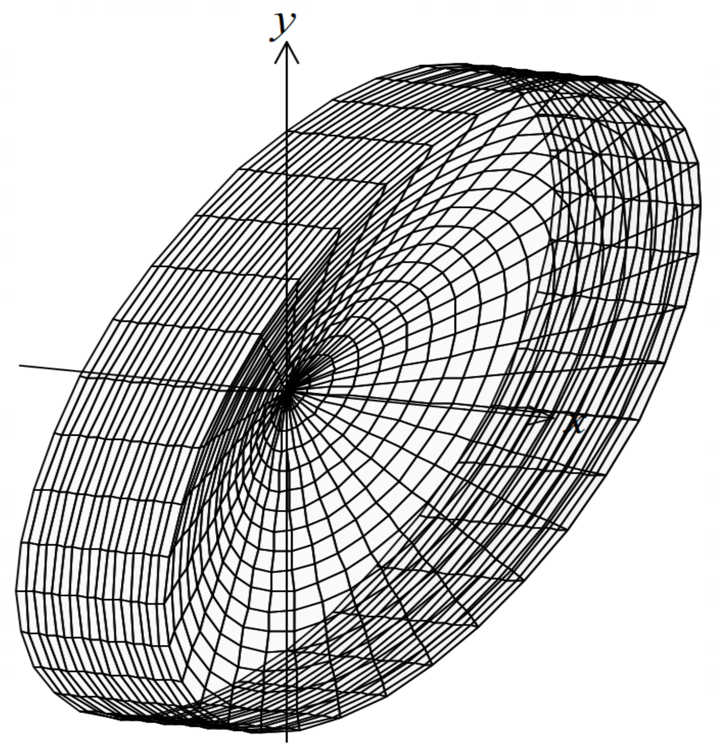
$$= \pi \int_0^1 \left\{5^2 - (5x)^2\right\} dx$$

$$= \pi \int_0^1 \left(25 - 25x^2\right) dx$$

$$= 25\pi \left[x - \frac{1}{3}x^3\right]_0^1$$

$$= 25\pi \left\{1 - \frac{1}{3}(1)^3 - 0\right\}$$

$$= \frac{50\pi}{3} \text{ unit}^3$$

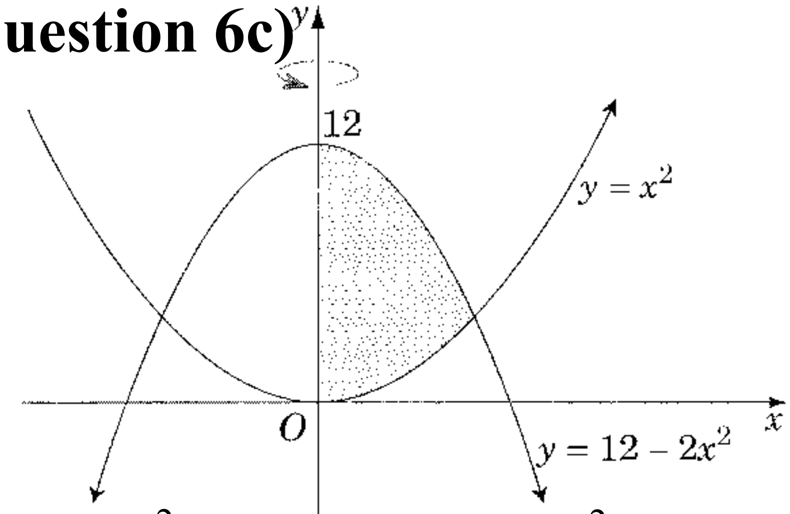


OR $V = \pi r^2 h - \frac{1}{3}\pi r^2 h$

$$= \frac{2}{3}\pi(5)^2(1)$$

$$= \frac{50\pi}{3} \text{ units}^3$$

2005 Advanced HSC Question 6c)



The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram.

(i) Find the points of intersection of the two curves. **(1)**

$$x^2 = 12 - 2x^2$$

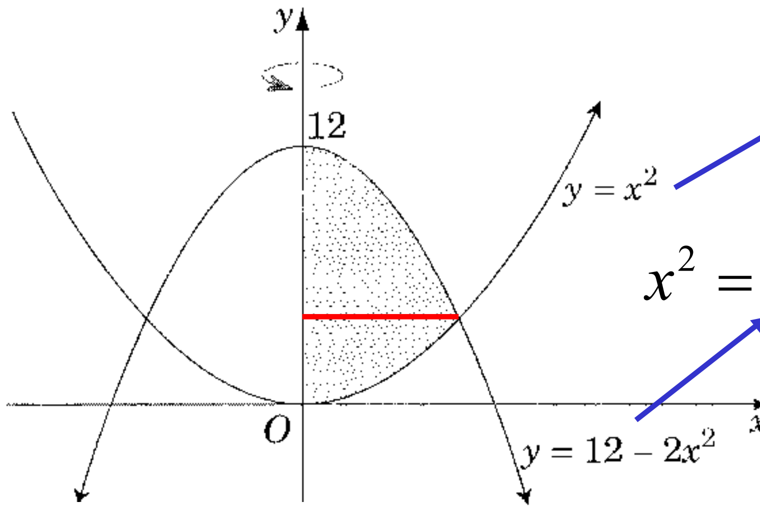
$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

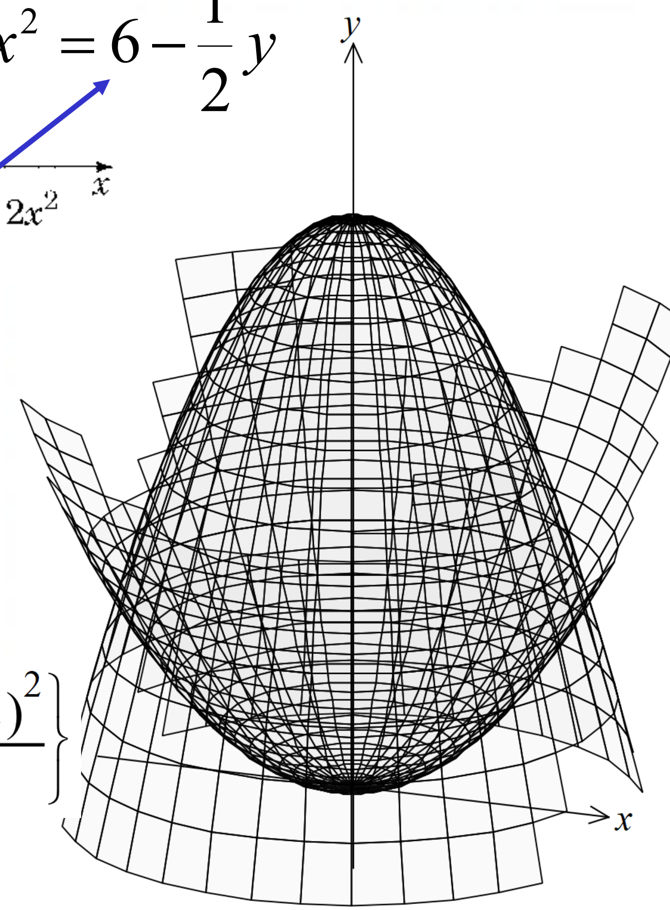
$$\therefore \text{ meet at } (2, 4)$$

(ii) The shaded region between the two curves and the *y* axis is rotated about the *y* axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed. **(3)**



$$x^2 = y$$

$$x^2 = 6 - \frac{1}{2}y$$



$$V = \pi \int x^2 dy$$

$$V = \pi \int_0^4 y\,dy + \pi \int_4^{12} \left(6 - \frac{1}{2}y\right) dy$$

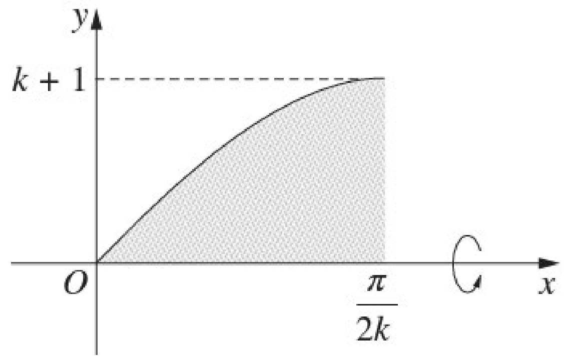
$$= \pi \left[\frac{y^2}{2}\right]_0^4 - \pi \left[6y - \frac{y^2}{4}\right]_{12}^4$$

$$= \pi\left\{\frac{(4)^2}{2} - 6(4) + \frac{(4)^2}{4}\right\} - \pi\frac{(0)^2}{2} + \pi\left\{6(12) - \frac{(12)^2}{4}\right\}$$

$$= \underline{24\pi \text{ units}^2}$$

2022 Extension 1 HSC Question 13b)

A solid of revolution is to be found by rotating the region bounded by the x-axis and the curve $y = (k + 1)\sin(kx)$, where $k > 0$, between $x = 0$ and $x = \dfrac{\pi}{2k}$ about the x-axis



Find the value of k for which the volume is π^2

$$V = \pi\int_0^{\frac{\pi}{2k}} (k+1)^2\sin^2(kx)\,dx$$

$$V = \pi \int_{0}^{\frac{\pi}{2k}} (k+1)^2 \sin^2(kx)\,dx$$

$$= \frac{\pi(k+1)^2}{2} \int_{0}^{\frac{\pi}{2k}} (1 - \cos 2kx)\,dx$$

$$= \left(\pi(k+1)^2\right)\left[x - \frac{1}{2k}\sin 2kx\right]_{0}^{\frac{\pi}{2k}}$$

$$= \left(\pi(k+1)^2\right)\left(\frac{\pi}{2k} - 0 - 0 + 0\right)$$

$$= \frac{\pi^2(k+1)^2}{4k}$$

$$V = \pi^2$$

$$\frac{\pi^2(k+1)^2}{4k} = \pi^2$$

$$(k+1)^2 = 4k$$

$$k^2 + 2k + 1 = 4k$$

$$k^2 - 4k + 1 = 0$$

$$(k-1)^2 = 0$$

$$\underline{k = 1}$$

Exercise 12F; 3bdef, 4eg, 6, 9, 10cd, 11d, 13, 14, 16ad, 18, 20, 21, 23, 24, 25