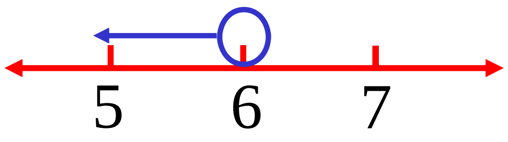
Inequations

An **inequation** is a problem where we are trying to find possible values, solved the same as an equation, ending up with a pronumeral as the subject of the inequation.

> The inequality sign will only change when:
>
> 1) Multiply or divide by a negative number
>
> *"if you change the sign, you change the sign"*
>
> 2) The reciprocal of both sides are taken

e.g. $(i)\ 6x < 36$

$$x < 6$$

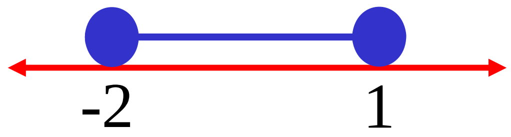


$(ii)\ 2 \leq 6 - 4x \leq 14$

$$-4 \leq -4x \leq 8$$

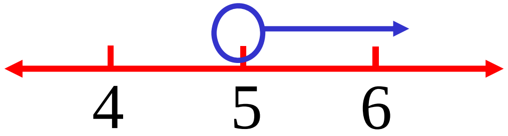
$$1 \geq x \geq -2$$

$$-2 \leq x \leq 1$$



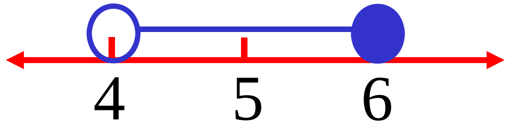
The "*correct*" way of writing inequalities

the algebraic solution should match (look like) the geometrical solution



$x > 5$

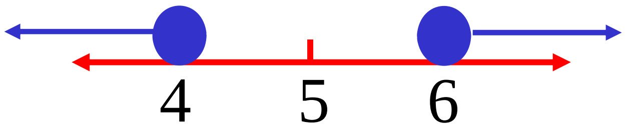
NOT $5 < x$



$4 < x \leq 6$

NOT $6 \geq x > 4$

NOT $x > 4$ or $x \leq 6$



$x \leq 4$ or $x \geq 6$

NOT $x \geq 6$ or $x \leq 4$

NOT $4 \geq x \geq 6$

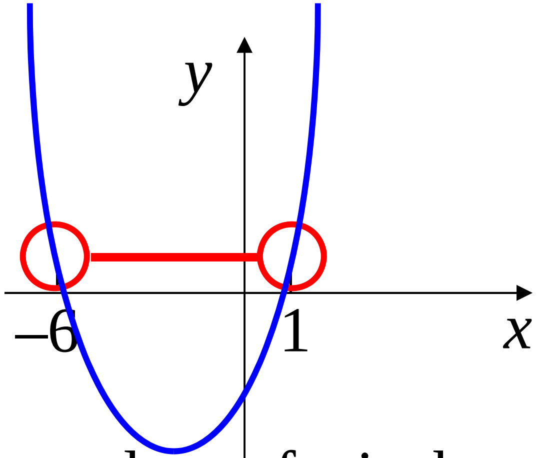
Quadratic Inequations

e.g. (*i*) $x^2 + 5x - 6 < 0$

$$(x+6)(x-1) < 0$$

$$-6 < x < 1$$

The answer is the domain of the function, when the range is restricted to $y < 0$



Q: for what values of x is the parabola below the x axis?

(*ii*) $-x^2 - 3x + 4 \le 0$

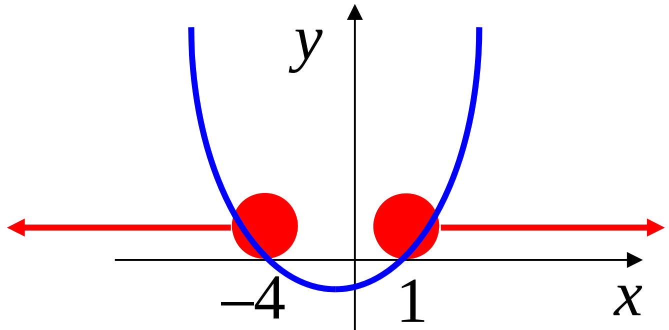
$$x^2 + 3x - 4 \ge 0$$

$$(x+4)(x-1) \ge 0$$

$$x \le -4 \text{ or } x \ge 1$$

Note: *quadratic inequalities always have solutions in the form*

? < x < ? *OR* *x < ? or x > ?*



Q: for what values of x is the parabola above the x axis?

To solve an inequation;

1. Solve the **equation**
2. Test the regions

Absolute Value Inequations

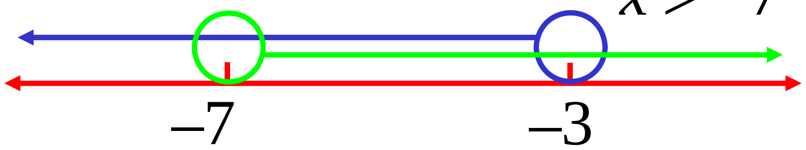
e.g. $(i)\ \ |x+5|<2$

$$x+5<2 \quad \text{or} \quad -(x+5)<2$$

$$x<-3 \qquad\qquad -x-5<2$$

$$-x<7$$

$$x>-7$$



$$\therefore -7<x<-3$$

$(ii)\ \ |3x+2|\geq 1$

$$3x+2\geq 1 \quad \text{or} \quad -(3x+2)\geq 1$$

$$3x\geq -1 \qquad\qquad -3x-2\geq 1$$

$$x\geq -\frac{1}{3} \qquad\qquad -3x\geq 3$$

$$x\leq -1$$

$$\therefore x\leq -1 \quad \text{or} \quad x\geq -\frac{1}{3}$$

alternate method: turn it into a quadratic inequation

e.g. $(i)\ |x+5|<2$

1) square both sides

(squares are always positive, just like absolute value)

2) Solve the quadratic inequation

$$(x+5)^2<4$$

$$x^2+10x+25<4$$

$$x^2+10x+21<0$$

$$(x+7)(x+3)<0$$

$$\therefore -7<x<-3$$

$(ii)\ |3x+2|\geq 1$

$$(3x+2)^2\geq 1$$

$$9x^2+12x+4\geq 1$$

$$9x^2+12x+3\geq 0$$

$$3x^2+4x+1\geq 0$$

$$(3x+1)(x+1)\geq 0$$

$$\therefore x\leq -1 \quad \text{or} \quad x\geq -\frac{1}{3}$$

Inequations with Pronumerals in the Denominator

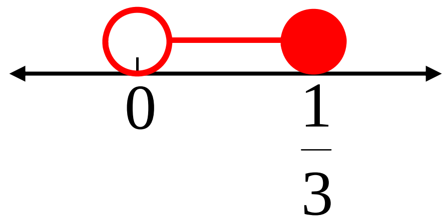
e.g. $(i)\ \frac{1}{x} \geq 3$ 1) Find the value where the denominator is zero $x \neq 0$

2) Solve the "equation"

$$\frac{1}{x} = 3$$

$$x = \frac{1}{3}$$

3) Plot these values on a number line



4) Test regions

Test $x = -1$ $\frac{1}{-1} \geq 3$ ✗ Test $x = \frac{1}{4}$ $\frac{1}{\frac{1}{4}} \geq 3$ ✓ Test $x = 1$ $\frac{1}{1} \geq 3$ ✗

$$\therefore 0 < x \leq \frac{1}{3}$$

$(ii)\ \frac{2}{x+3} < 5$

$$x + 3 \neq 0$$

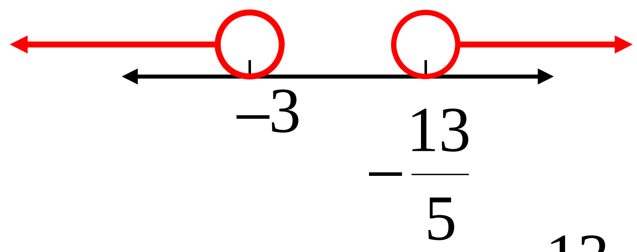
$$x \neq -3$$

$$\frac{2}{x+3} = 5$$

$$2 = 5x + 15$$

$$5x = -13$$

$$x = -\frac{13}{5}$$



$$\therefore x < -3 \text{ or } x > -\frac{13}{5}$$

alternate method: turn it into a quadratic inequation

e.g. $(i)\ \frac{1}{x} \geq 3$

1) multiply both sides by the denominator squared

(to ensure it is a positive number, so the sign stays the same)

$$\frac{1}{x} \times x^2 \geq 3x^2$$

2) Solve the quadratic inequation

$$x \geq 3x^2$$

3) Take care of the domain issue

(bottom of fraction cannot equal zero)

$$3x^2 - x \leq 0$$

$$x(3x-1) \leq 0$$

$$0 \leq x \leq \frac{1}{3} \quad , x \neq 0$$

$$\therefore 0 < x \leq \frac{1}{3}$$

Exercise 5A; 1jl, 2d, 3ace, 4, 9ace, 11bdf, 12bdf, 13f, 14ac, 15ab(ii), 17, 18 , 19, 20

$$(ii)\ \frac{2}{x+3} < 5$$

$$2(x+3) < 5(x+3)^2$$

$$5(x+3)^2 - 2(x+3) > 0$$

$$(x+3)(5x+13) > 0$$

$$\therefore x < -3 \text{ or } x > -\frac{13}{5} \quad , x \neq -3$$

$$\therefore x < -3 \text{ or } x > -\frac{13}{5}$$