Integration Strategies

(1) look for an obvious solution

Try to find some function $u = g(x)$ in the integrand whose derivative also occurs

e.g $$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx$$

$$= -\sqrt{1-x^2} + c$$

$$\int \frac{du}{\sqrt{u}} = 2\sqrt{u}$$

not on the reference sheet, but worth remembering

(2) manipulation into a standard form

Standard Integrals are designed to save time when integrating. Once the integrand has been manipulated into a standard form, then the primitive function can simply be quoted.

e.g. (i) $$\int e^{x+e^x}\,dx = \int e^x\, e^{e^x}\,dx$$

$$= e^{e^x} + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

(ii) $$\int \frac{2e^{\frac{x}{2}}}{3e^x + 6} dx = \frac{4}{3} \int \frac{\frac{1}{2} e^{\frac{x}{2}}}{2 + \left[e^{\frac{x}{2}} \right]^2} dx$$

$$= \frac{4}{3} \times \frac{1}{\sqrt{2}} \tan^{-1} \frac{e^{\frac{x}{2}}}{\sqrt{2}} + c$$

$$= \frac{2\sqrt{2}}{3} \tan^{-1} \frac{e^{\frac{x}{2}}}{\sqrt{2}} + c$$

(3) simplify the integrand, if possible

The use of algebraic manipulation or trig identities may make the method of integration more obvious.

e.g. (*i*) $$\int \frac{\tan\theta}{\sec^2\theta} d\theta = \int \frac{\sin\theta}{\cos\theta} \times \frac{\cos^2\theta}{1} d\theta$$

$$= \int \sin\theta\cos\theta d\theta$$

$$= \frac{1}{2}\int \sin 2\theta d\theta$$

$$= -\frac{1}{4}\cos 2\theta + c$$

(ii) $\int x^3\sqrt{4-x^2}\,dx$

$$= \int \left[-x\left(4-x^2\right)+4x\right]\sqrt{4-x^2}\,dx$$

$$= \int \left[-x\left(4-x^2\right)^{\frac{3}{2}}+4x\left(4-x^2\right)^{\frac{1}{2}}\right]dx$$

$$= \frac{1}{2}\times\frac{2}{5}\left(4-x^2\right)^{\frac{5}{2}}-2\times\frac{2}{3}\left(4-x^2\right)^{\frac{3}{2}}+c$$

$$= \frac{1}{5}\left(4-x^2\right)^2\sqrt{4-x^2}-\frac{4}{3}\left(4-x^2\right)\sqrt{4-x^2}+c$$

factorise by using the expression in the grouping symbols, a multiple of its derivative or a combination of both

Exercise 4A; 1ef, 2ae, 3cf, 4ef, 5ce, 6, 7, 8, 9

Exercise 4B: 1c, 2c, 3bd, 4, 5, 6, 7ac, 8, 9, 10

(4) classify the integrand according to its form

- rational functions; use polynomial division and partial fractions
- trig functions; the power of the trig function will determine the technique to be used
- mixture of function types; try integration by parts

there are basically only two methods; substitution and by parts