Asymptotes

Curves always bend towards the asymptotes

Curves never cross a **vertical** asymptote

Curves approach **horizontal** and **oblique** asymptotes as $x \to \pm\infty$

solve $R(x) = 0$ to find where (if anywhere) the curve cuts the horizontal/oblique asymptote

$$y = \frac{P(x)}{A(x)} = Q(x) + \frac{R(x)}{A(x)}$$

$y = Q(x)$ is the horizontal/oblique asymptote

solve $A(x) = 0$ to find vertical asymptotes e.g. $(i)\ y=\frac{(x+3)(x-2)}{(x-1)(x+1)}$

$$\begin{array}{r|l} & 1 \\ \hline x^2-1 & x^2+x-6 \\ & x^2 \quad\ -1 \\ \hline & \quad\ x-5 \end{array}$$

$$y=1+\frac{x-5}{(x-1)(x+1)}$$

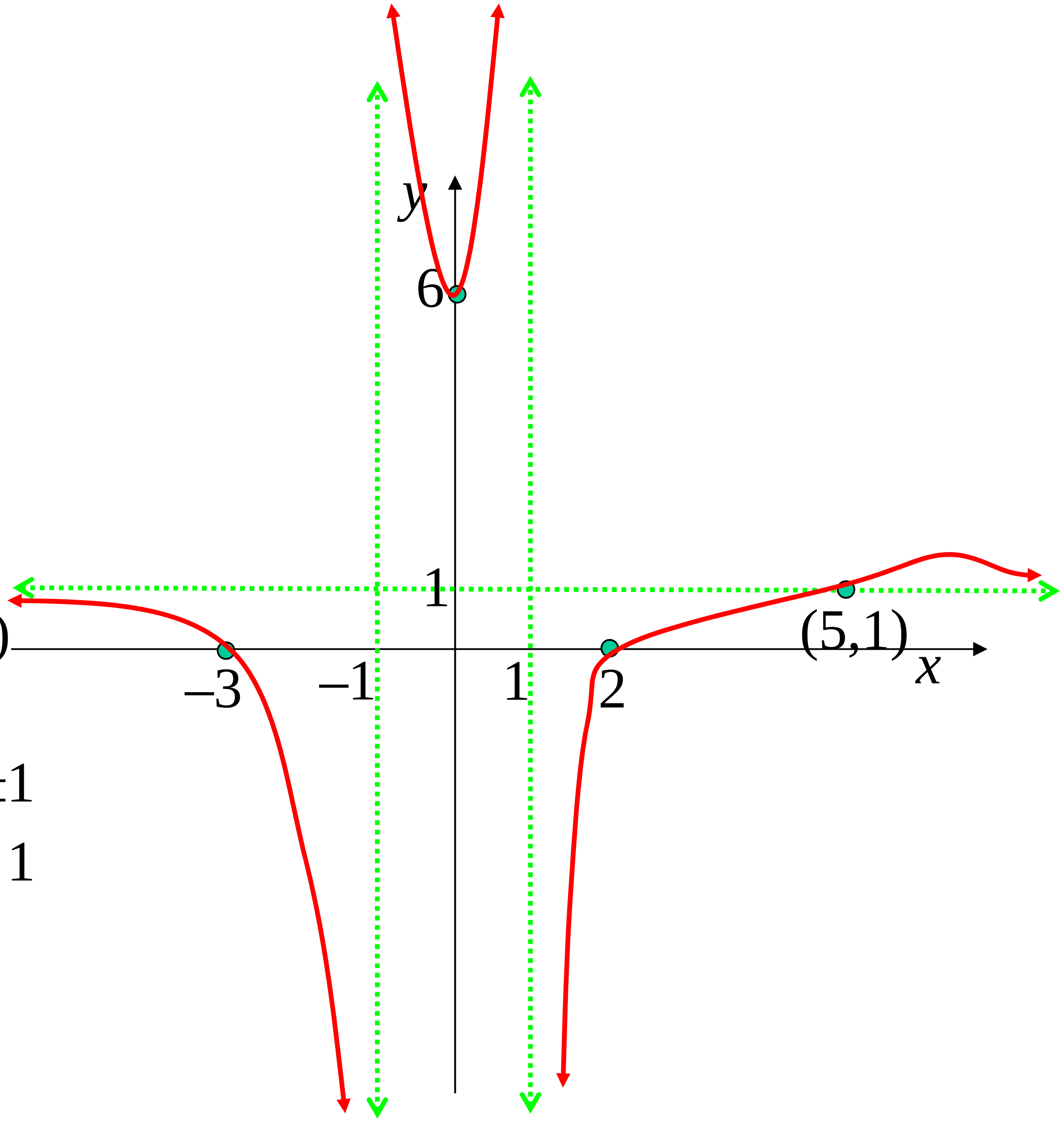
x intercepts: (–3,0) , (2,0)

y intercept: (0,6)

vertical asymptotes: $x=\pm 1$

horizontal asymptote: $y=1$

cuts horizontal asymptote at $x=5$



$$(ii)\ y=\frac{(x-2)(x-1)(x+1)}{(x+2)(x-3)}$$

$$\begin{array}{r|l} & x \quad -1 \\ \hline x^2-x-6 & x^3-2x^2-x+2 \\ & \underline{x^3-x^2-6x} \\ & \quad -x^2+5x+2 \\ & \quad \underline{-x^2+x\ +6} \\ & \qquad 4x-4 \end{array}$$

$$y=x-1+\frac{4x-4}{(x+2)(x-3)}$$

x intercepts: (–1,0), (1,0), (2,0)

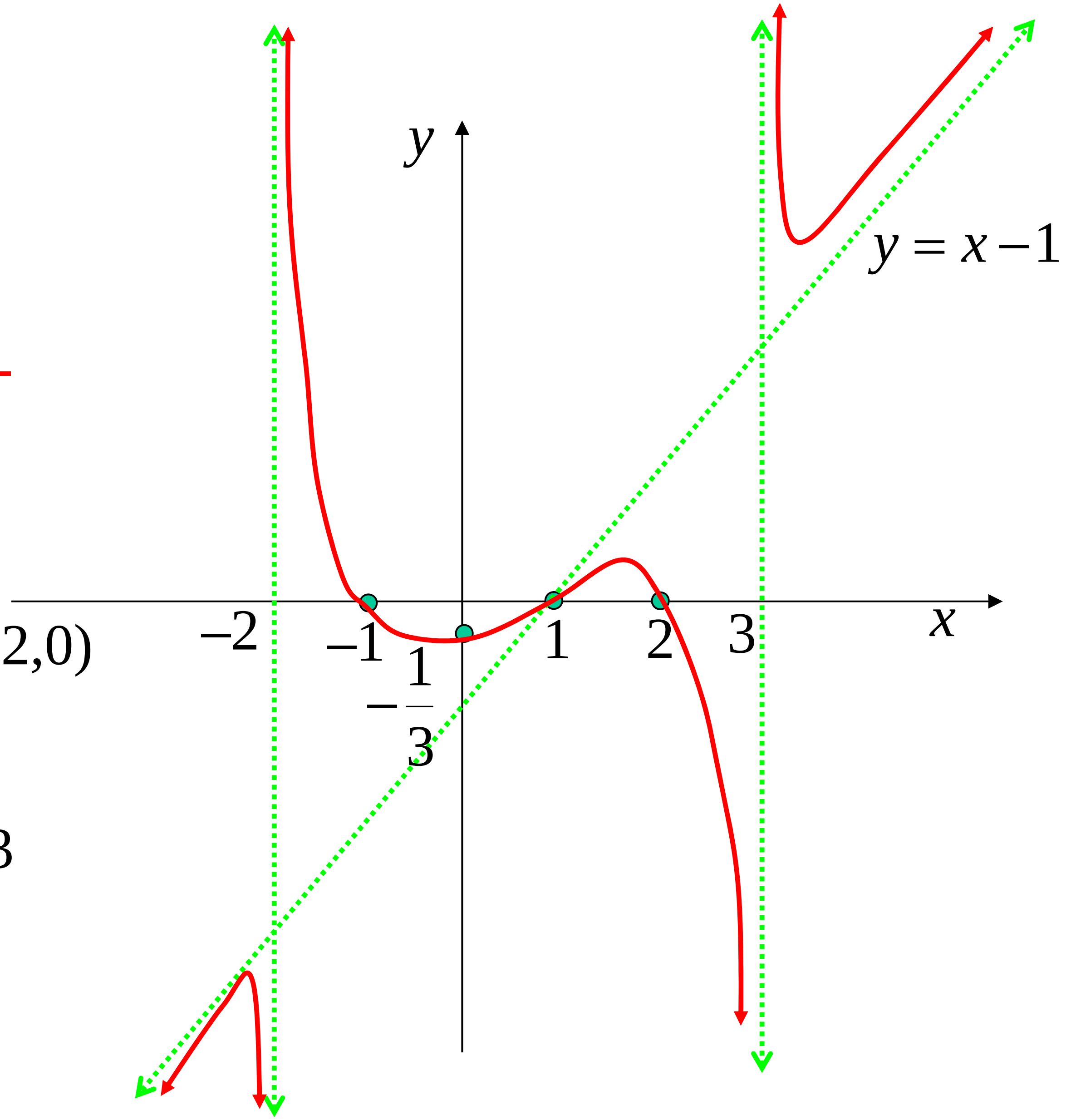
y intercept: $\left(0,-\frac{1}{3}\right)$

vertical asymptotes: $x=-2,3$

oblique asymptote: $y=x-1$

cuts horizontal

asymptote at $x=1$



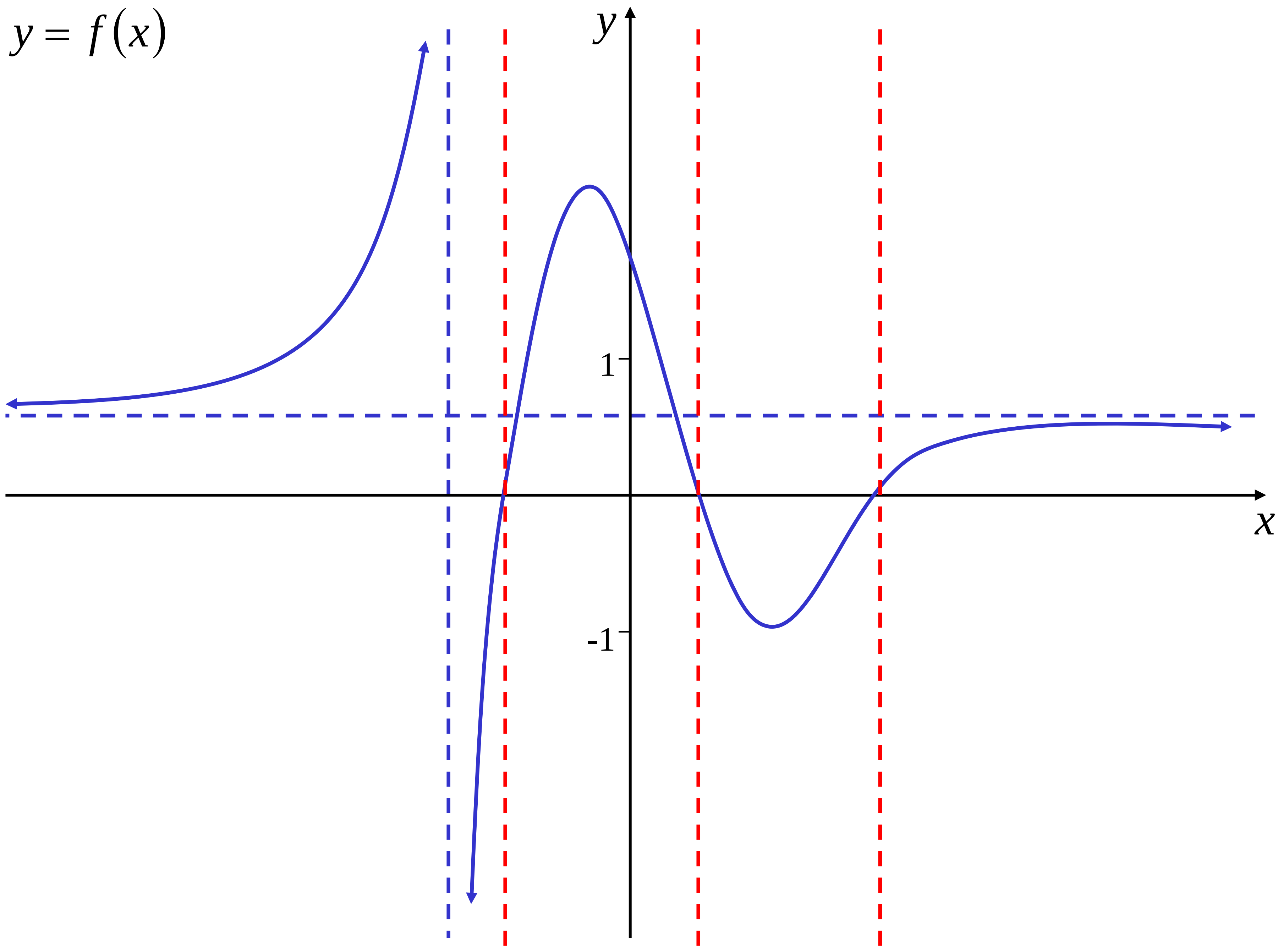
Graphs of Reciprocal Functions

The graph of $y = \dfrac{1}{f(x)}$ can be sketched by first drawing $y = f(x)$

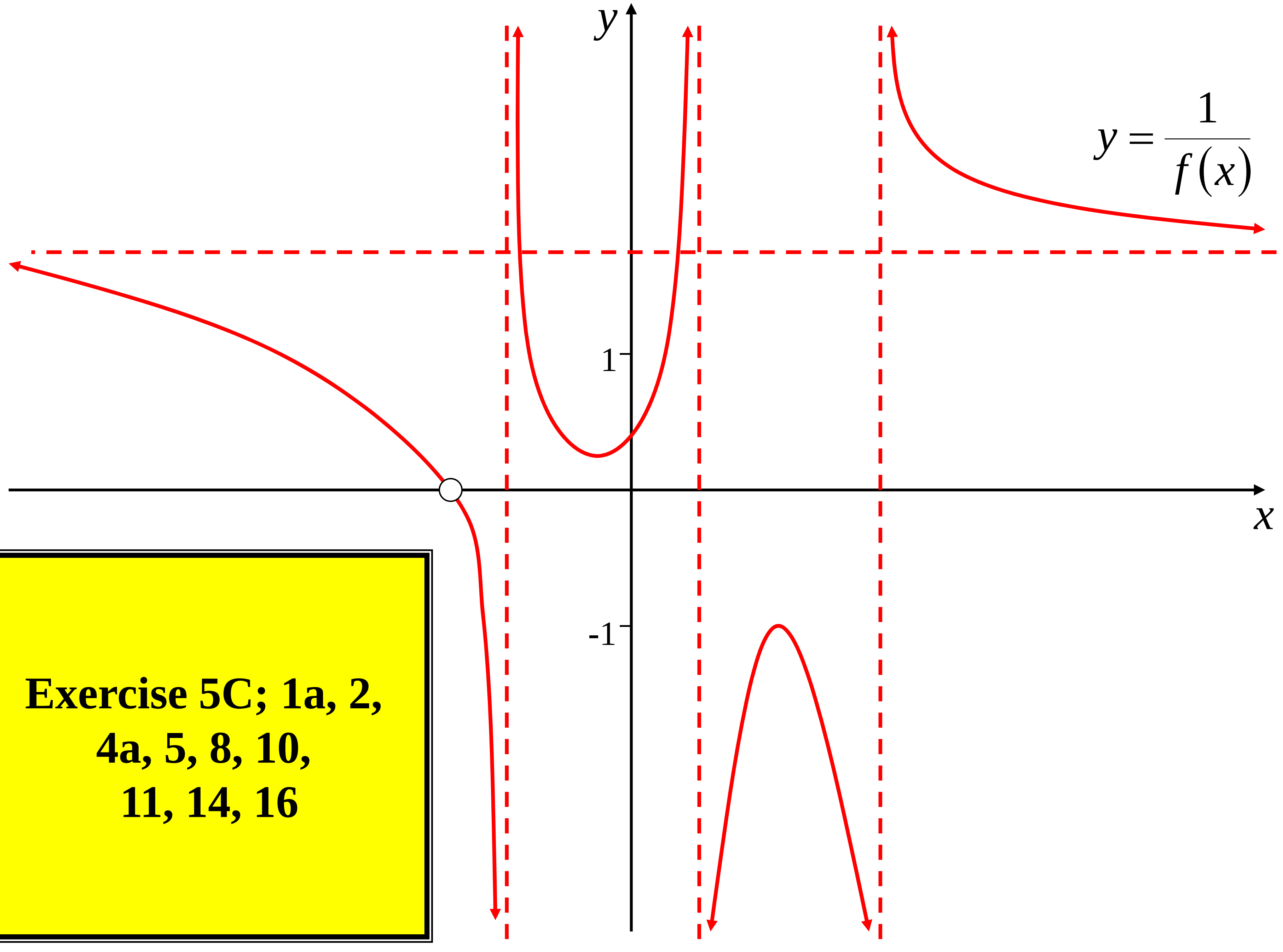
and noticing;

- when $f(x) = 0$, then $\dfrac{1}{f(x)}$ is undefined, (i.e. a vertical asymptote exists)
- when $f(x) \to \infty$, then $\dfrac{1}{f(x)} \to 0$, (i.e. asymptotes become x intercepts)
- when $f(x)$ is increasing, the reciprocal is decreasing, and visa - versa
- when $f(x)$ is positive, $\dfrac{1}{f(x)}$ is positive, etc.
- the derivative of $\dfrac{1}{f(x)}$ is $\dfrac{-f'(x)}{[f(x)]^2}$, hence stationary points of the

  original curve are stationary points of its reciprocal.



y = f(x)
y
1
-1
x



Exercise 5C; 1a, 2, 4a, 5, 8, 10, 11, 14, 16