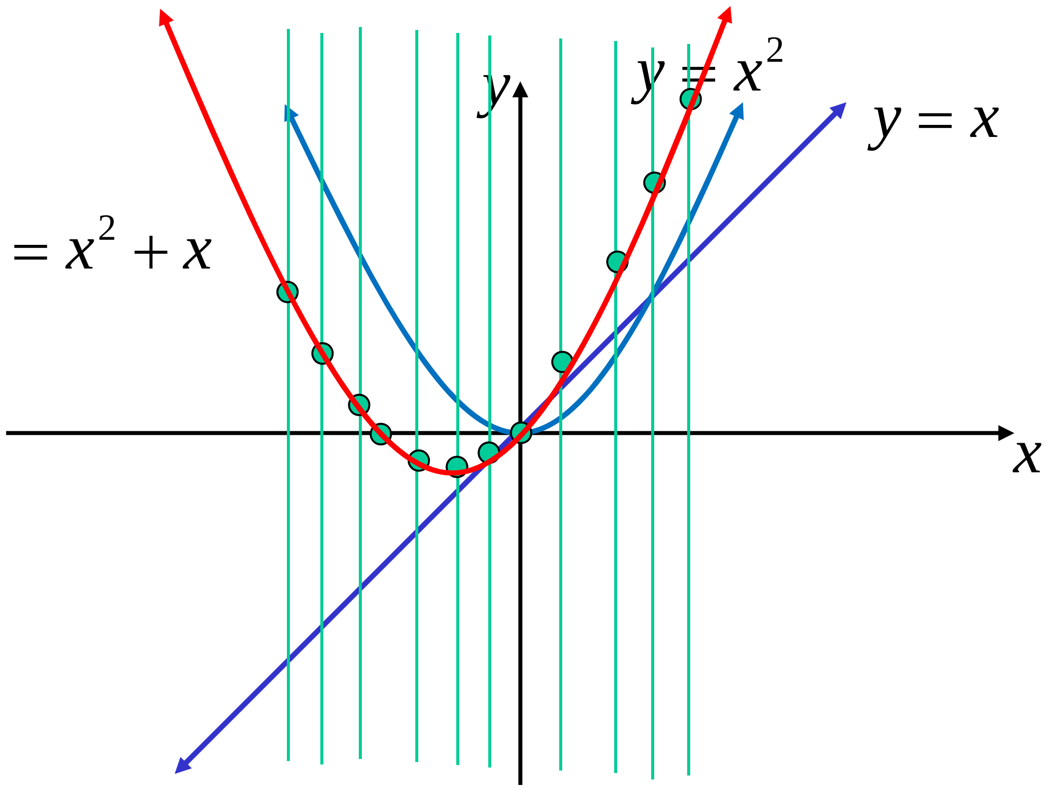
Addition of Graphs

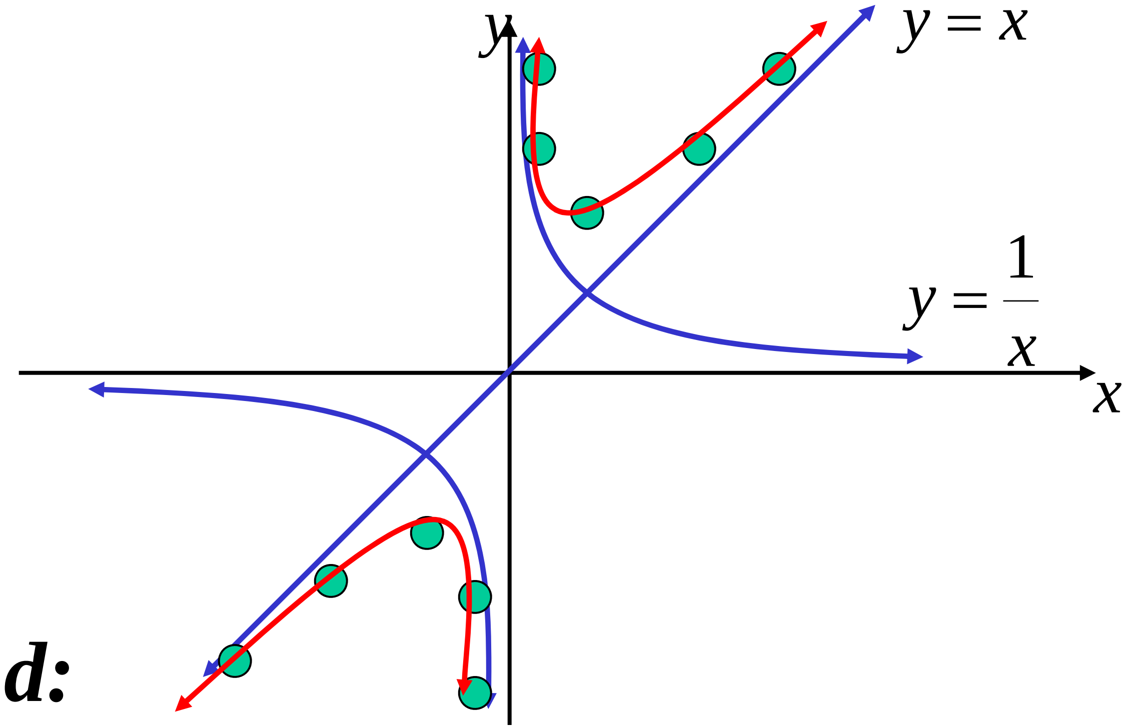
$y = f(x) + g(x)$ can be graphed by first graphing $y = f(x)$ and $y = g(x)$ separately and then adding their ordinates together.

- find and mark the x and y intercepts
- draw lines perpendicular to x axis cutting both curves
- add the y coordinates along each line and mark the point
- join the points

e.g. $y = x^2 + x$



$$(ii)\ y = x + \frac{1}{x}$$

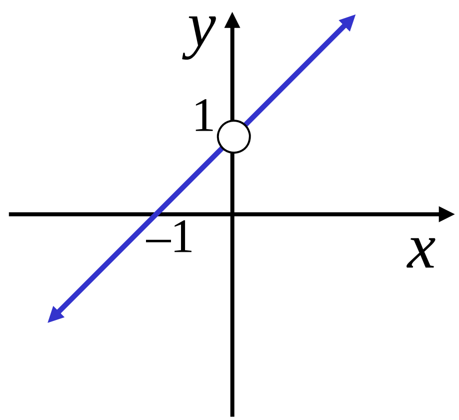


Things to keep in mind:

Discontinuities: any exclusions in the domain of the original function(s) remain in the new function

e.g. $f(x) = x + \frac{1}{x}$, $g(x) = 1 - \frac{1}{x}$

$$y = f(x) + g(x)$$
$$= x + 1\ ,\ x \neq 0$$



x-intercept: If $f(x) = -g(x)$, then $y = f(x) + g(x) = 0$

symmetry: like functions retain symmetry when added

odd function + odd function = odd function

even function + even function = even function e.g. $y = |x+3| + |1-x|$

$x \le -3;$

$y = -(x+3) + (1-x)$

$y = -x - 3 + 1 - x$

$y = -2x - 2$

$-3 < x < 1;$

$y = (x+3) + (1-x)$

$y = x + 3 + 1 - x$

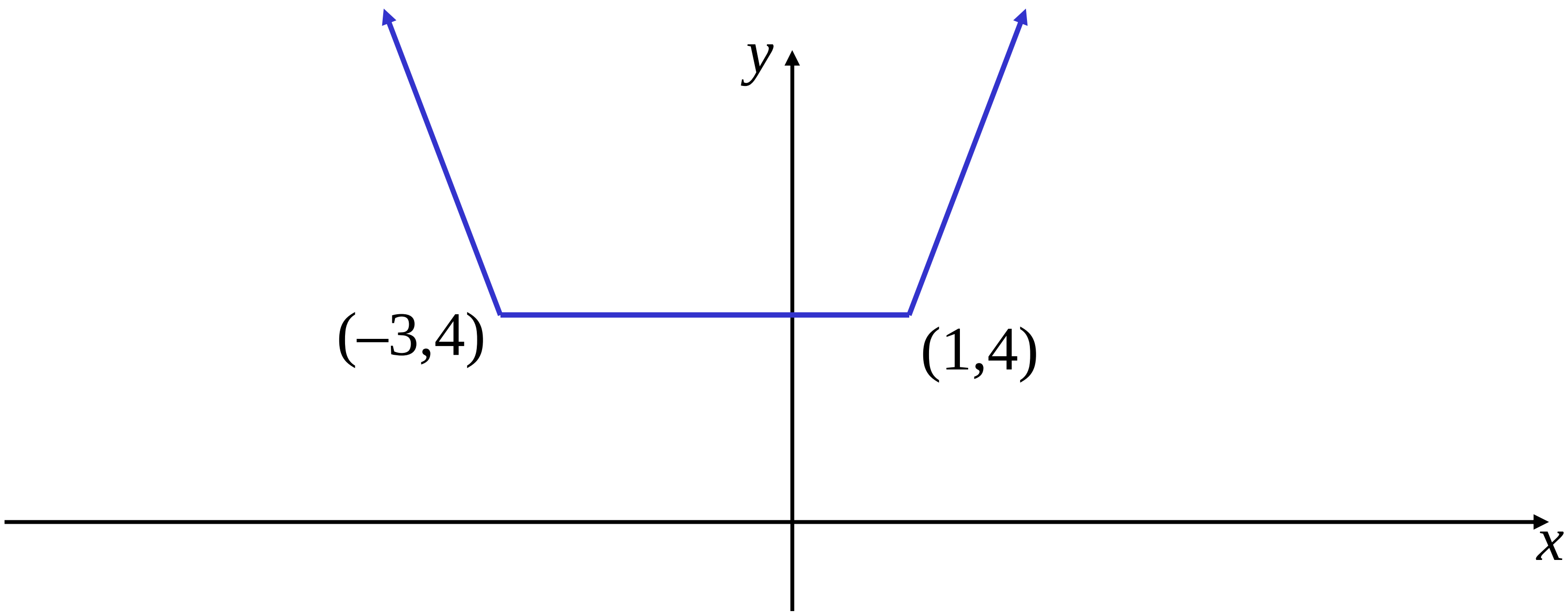
$y = 4$

$x \ge 1;$

$y = (x+3) - (1-x)$

$y = x + 3 - 1 + x$

$y = 2x + 2$

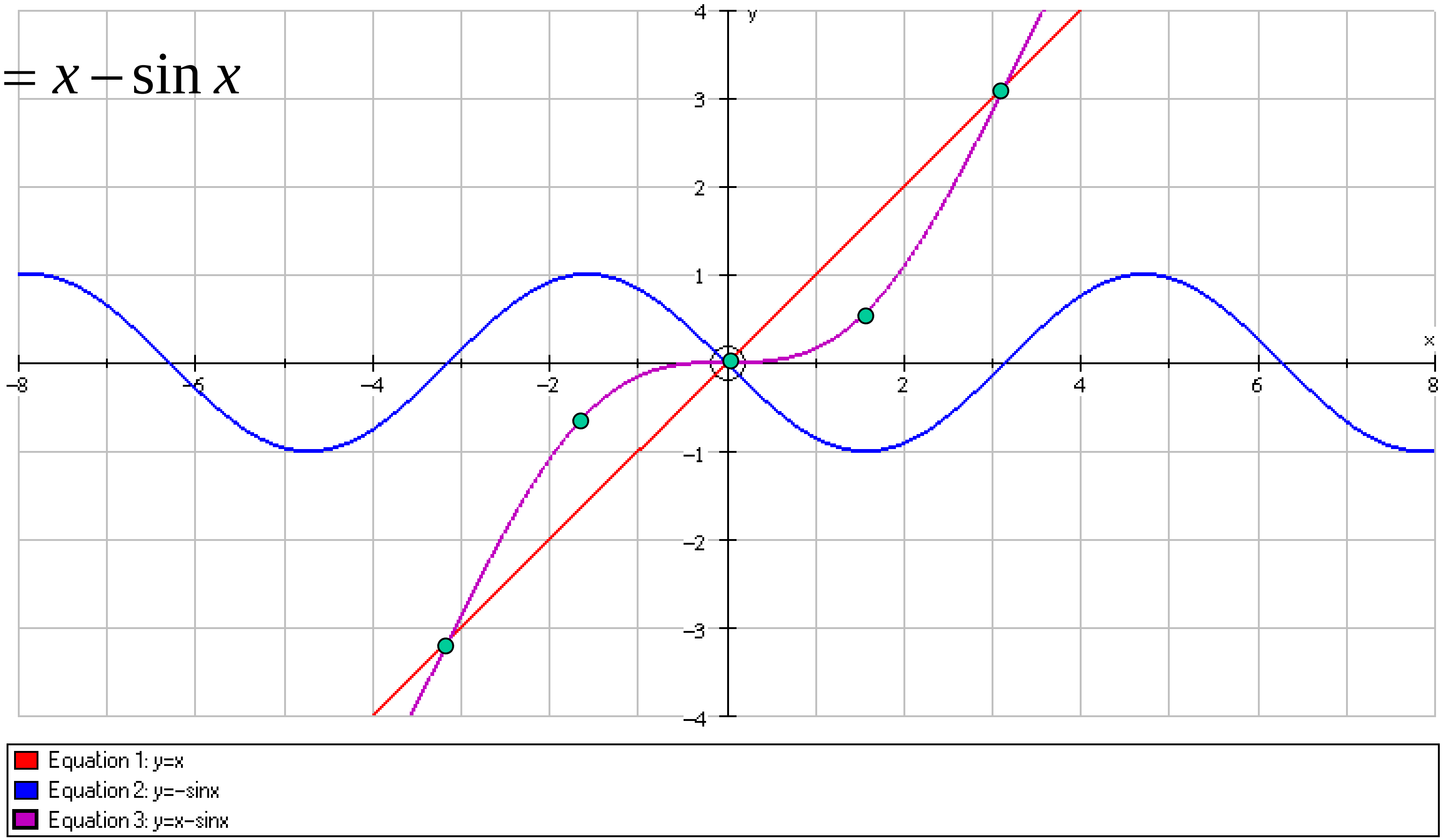


$y = f(x) - g(x)$ can be graphed by first graphing $y = f(x)$ and $y = -g(x)$ separately and then adding the ordinates together.

The graph of $y = x + g(x)$ where g(x) is bounded

If the graph of $y = g(x)$ is bounded by the lines $y = a$ and $y = b$, then $y = x + g(x)$ will be bounded by the lines $y = x + a$ and $y = x + b$

e.g. $y = x - \sin x$



Multiplication of Graphs

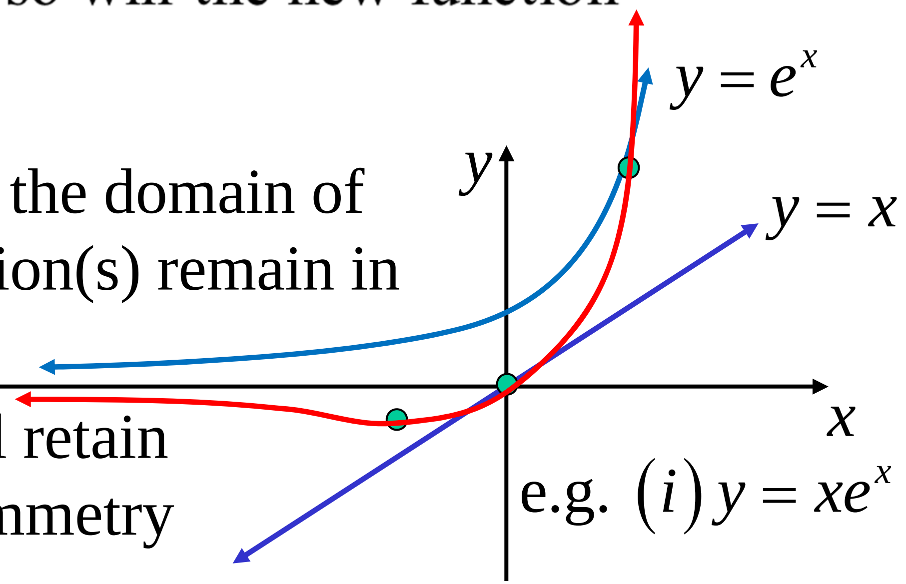
$y = f(x) \times g(x)$ can be graphed by first graphing $y = f(x)$ and $y = g(x)$ separately and then;

- mark the x intercepts, this will be where the new function changes sign
- multiply the "signs" of each function to determine the sign of the new function
- mark the y intercept
- special note needs to be made of points where $f(x) = 1$, or $g(x) = 1$ (and -1) .
- if $f(x)$ or $g(x) \to 0$ or $\pm\infty$, then so will the new function

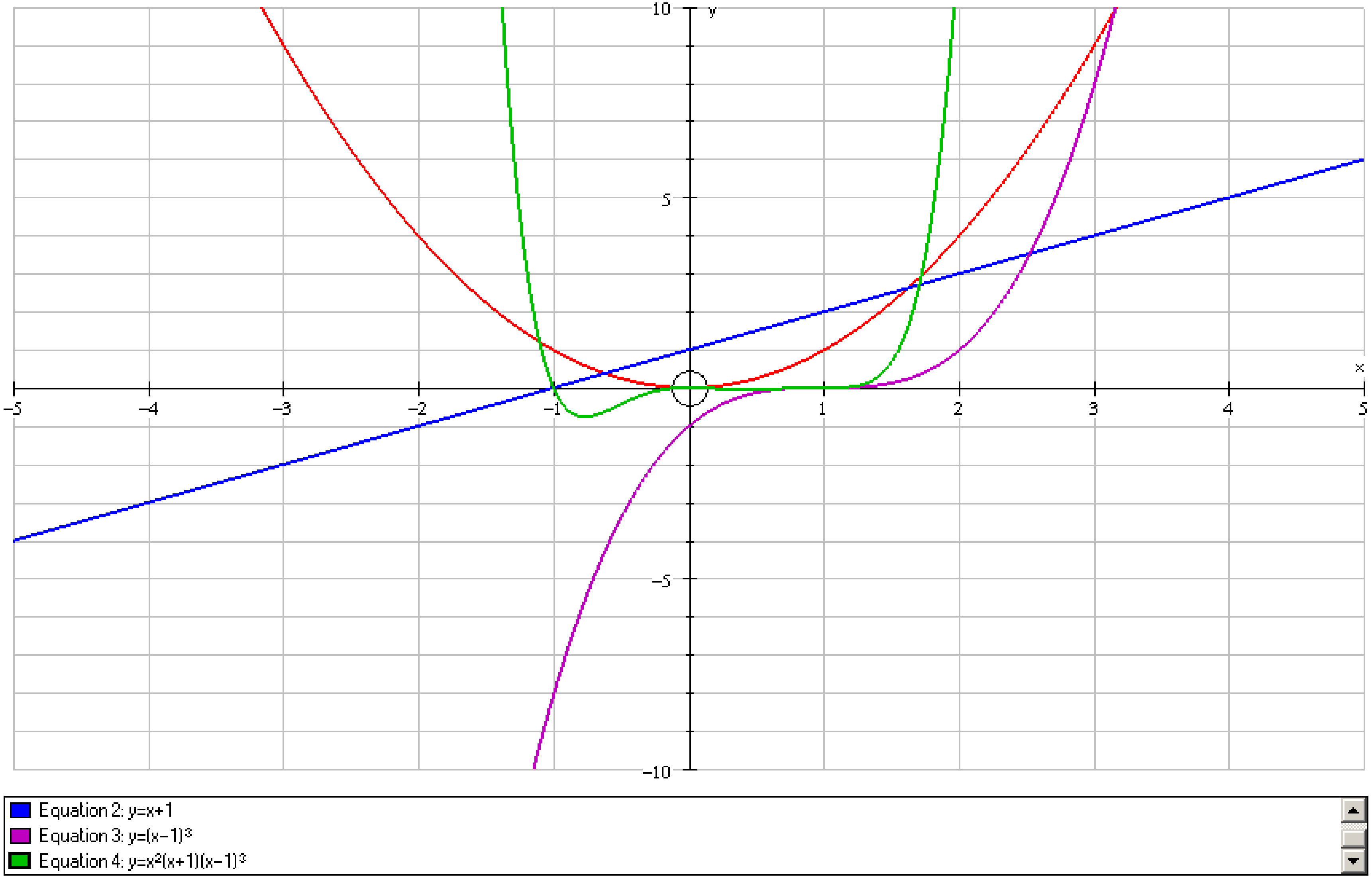
Things to keep in mind:

discontinuities: any exclusions in the domain of the original function(s) remain in the new function

symmetry: symmetric graphs will retain some form of symmetry



$$(ii)\ y = x^2(x+1)(x-1)^3$$

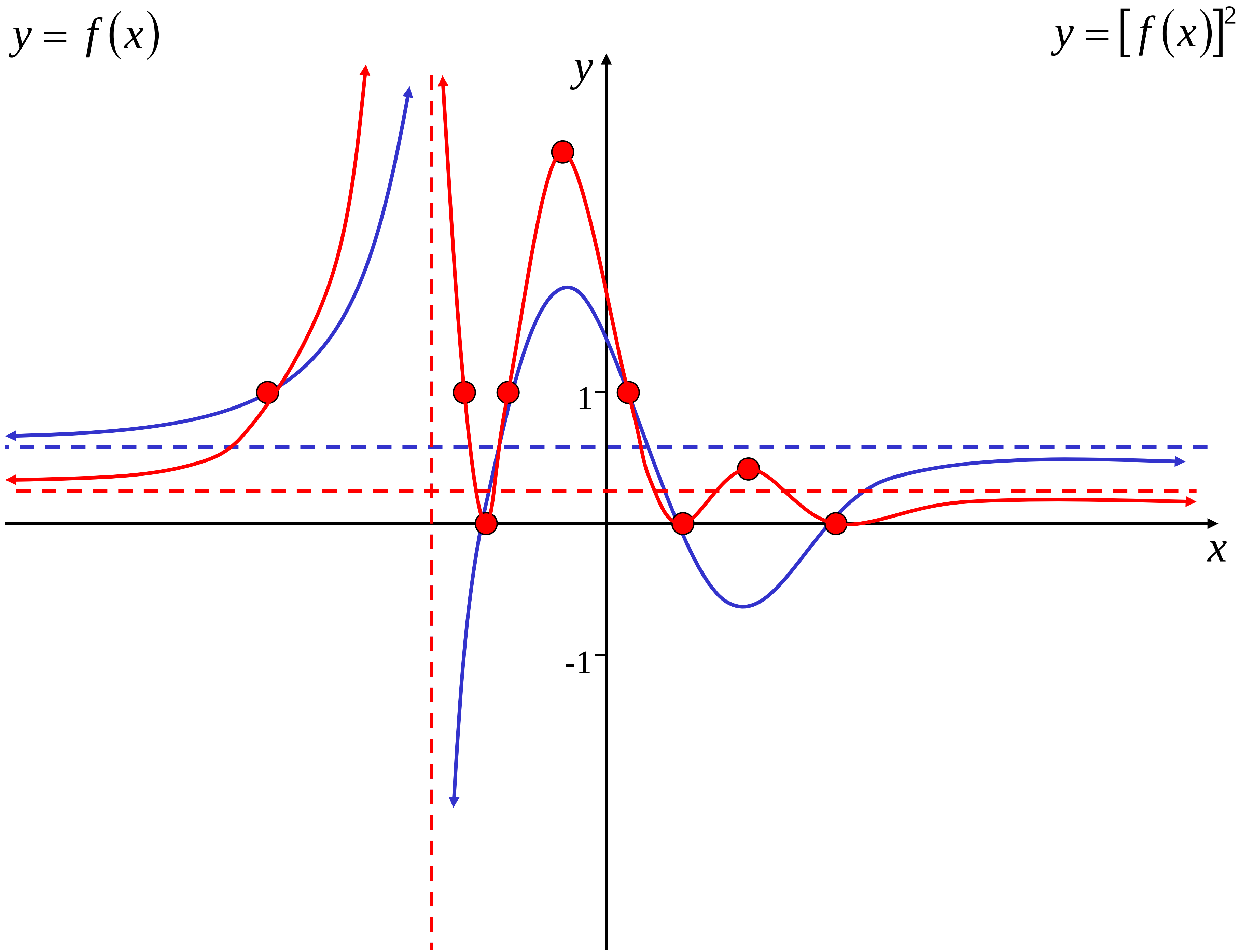


10
y
5
−5
−10
x
−5
−4
−3
−2
−1
1
2
3
4
5
Equation 2: y=x+1
Equation 3: y=(x−1)³
Equation 4: y=x²(x+1)(x−1)³

Graphs of the Form $y = \left[f(x) \right]^2$

$y = f(x) \times f(x)$ i.e. $y = [f(x)]^2$ can be graphed by first graphing $y = f(x)$ then;

- all single roots will become double roots
- all stationary points must still be stationary points
- all discontinuities will remain
- horizontal and oblique asymptotes may change (square their value)
- if $|f(x)| > 1$ then $[f(x)]^2 > f(x)$ i.e. new curve is above the old curve
- if $|f(x)| < 1$ then $[f(x)]^2 < f(x)$ i.e. new curve is below the old curve



$y = f(x)$
$y = [f(x)]^2$
y
x
1
-1

Division of Graphs

$y=\dfrac{f(x)}{g(x)}$ can be thought of as $y=f(x)\times\dfrac{1}{g(x)}$ and the same procedures as multiplication can be followed except;

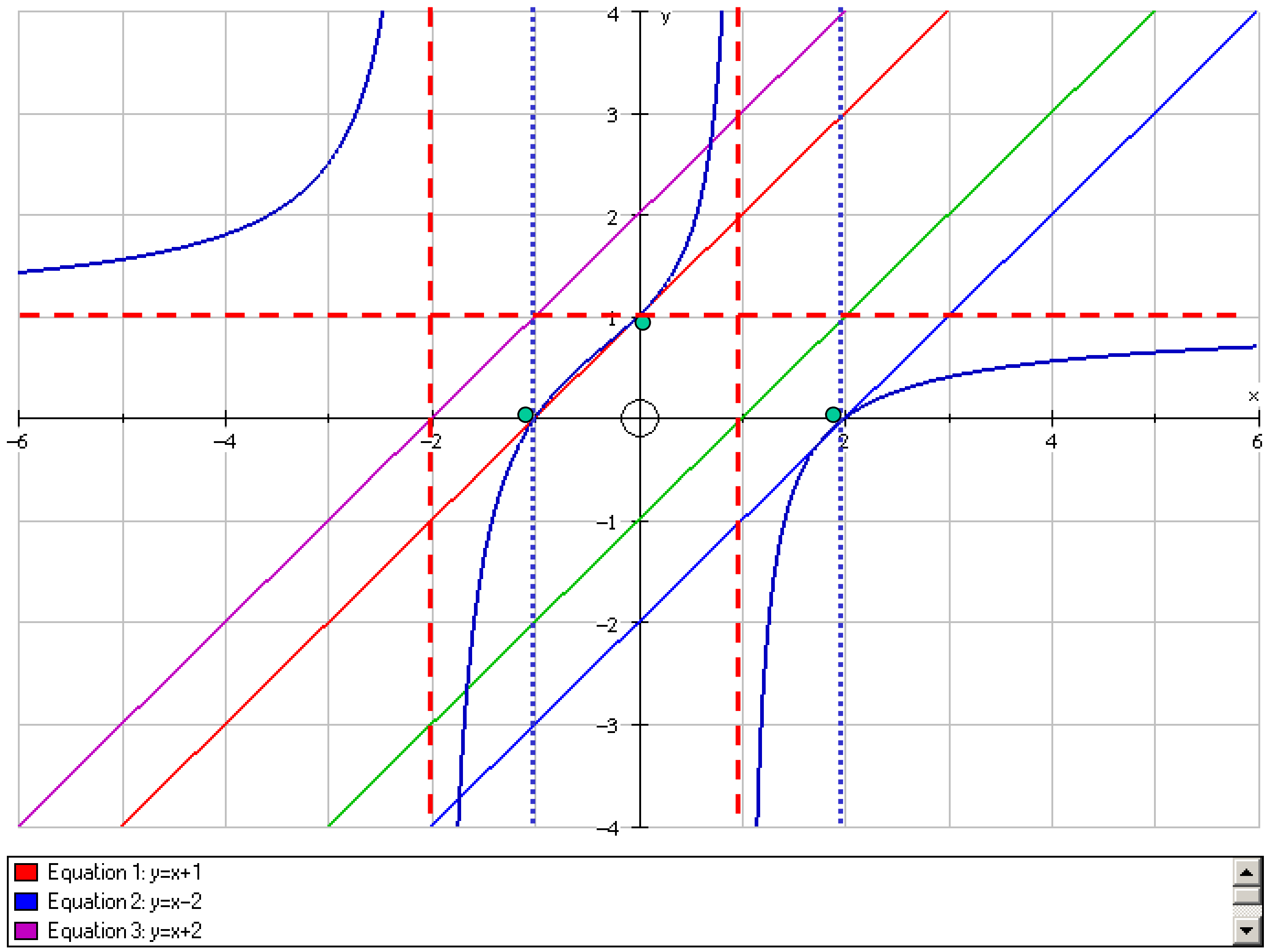
- the x intercepts of $g(x)$ will become vertical asymptotes or point discontinuities
- investigation the behaviour of the function for large values of x will be required (find horizontal/oblique asymptotes , look at dominance)

$$y=\frac{(x+1)(x-2)}{(x+2)(x-1)}$$

$$=\frac{x^2-x-2}{x^2+x-2}$$

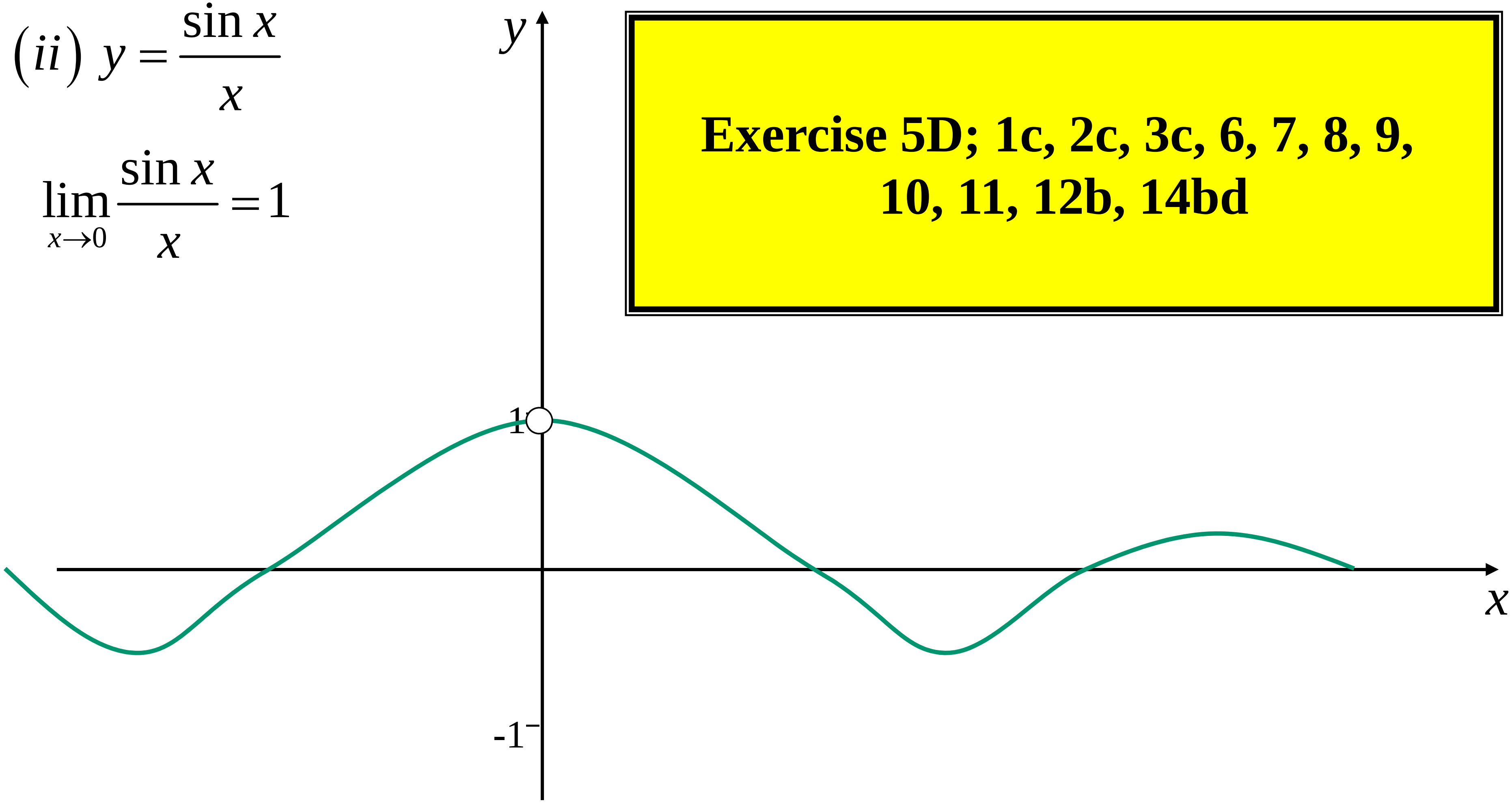
$$=1-\frac{2x}{x^2+x-2} \qquad \therefore \text{horizontal asymptote}: y=1$$

$$\text{e.g. } y = \frac{(x+1)(x-2)}{(x+2)(x-1)}$$



$$(ii)\ y = \frac{\sin x}{x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



Exercise 5D; 1c, 2c, 3c, 6, 7, 8, 9, 10, 11, 12b, 14bd

symmetry:

odd function × odd function = even function

even function × even function = even function

odd function × even function = odd function