Trigonometric Equations

e.g. $(i) \tan\theta = -2.3673$ $\qquad 0° \leq \theta \leq 360°$

Q2, Q4 $\longleftarrow$ locate quadrants

$\tan\alpha = 2.3673 \longleftarrow$ find acute angle

$\alpha = 67°6'$

$\theta = 180 - \alpha, 360 - \alpha$ $\qquad\longleftarrow$ solve the problem

$\theta = 180 - 67°6', 360 - 67°6'$

$\underline{\theta = 112°54', 292°54'}$

$(ii) \cos\theta = \dfrac{1}{2}$ $\qquad 0° \leq \theta \leq 360°$

Q1, Q4

$\cos\alpha = \dfrac{1}{2}$

$\theta = \alpha, 360 - \alpha$
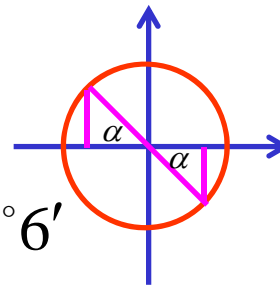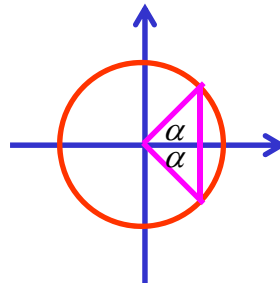
$\theta = 60°, 360 - 60°$

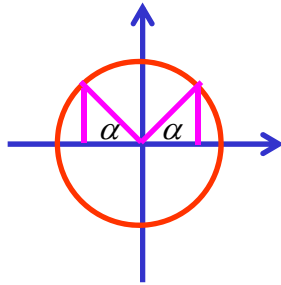$\alpha = 60°$

$\underline{\theta = 60°, 300°}$

$(iii)\,\text{cosec}\,\theta = 4$

$\sin\theta = \dfrac{1}{4}$

Q1, Q2

$\sin\alpha = \dfrac{1}{4}$

$\alpha = 14°29'$

$0° \le \theta \le 360°$

$\theta = \alpha, 180 - \alpha$

$\theta = 14°29', 180 - 14°29'$
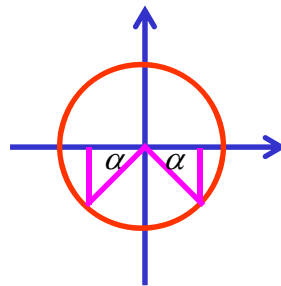
$\theta = 14°29', 165°31'$

$(iv)\,2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\dfrac{1}{2}$

Q3, Q4

$\sin\alpha = \dfrac{1}{2}$

$\alpha = 30°$

$0° \le x \le 360°$

$x = 180 + \alpha, 360 - \alpha$

$\theta = 180 + 30°, 360 - 30°$

$\theta = 210°, 330°$

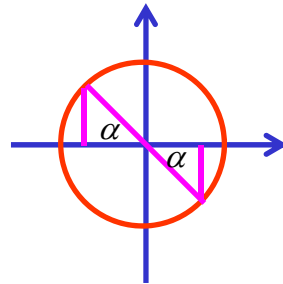$(v)\ 5\sin\theta = -2\cos\theta$ $0° \leq \theta \leq 360°$

$$\frac{\sin\theta}{\cos\theta} = \frac{-2}{5}$$

$$\tan\theta = \frac{-2}{5}$$

Q2, Q4

$$\tan\alpha = \frac{2}{5}$$

$$\alpha = 21°48'$$

$\theta = 180 - \alpha, 360 - \alpha$

$\theta = 180 - 21°48', 360 - 21°48'$

$\theta = 158°12', 338°12'$

$(vi)\ \cos 2\theta = 0.75$ $0° \leq \theta \leq 360°$

$0° \leq 2\theta \leq 720°$

Q1, Q4

$\cos\alpha = 0.75$
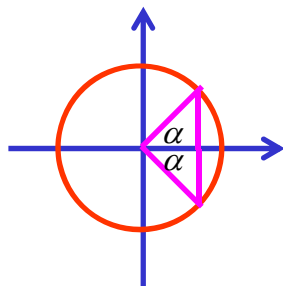
$\alpha = 41°25'$

$2\theta = \alpha, 360 - \alpha$

$2\theta = 41°25', 360 - 41°25'$

$2\theta = 41°25', 318°35', 401°25', 678°35'$

$\theta = 20°43', 159°18', 200°43', 339°18'$

$(vii)\sec^2\theta + \tan\theta = 3 \qquad 0° \le \theta \le 360°$

$1 + \tan^2\theta + \tan\theta = 3$

$\tan^2\theta + \tan\theta - 2 = 0$

$(\tan\theta + 2)(\tan\theta - 1) = 0$

$\tan\theta = -2 \qquad\qquad$ or $\qquad\qquad \tan\theta = 1$

Q2, Q4 $\qquad\qquad\qquad\qquad\qquad\qquad$ Q1, Q3

$\tan\alpha = 2 \qquad\qquad\qquad\qquad\qquad\qquad \tan\alpha = 1$

$\alpha = 63°26' \qquad\qquad\qquad\qquad\qquad \alpha = 45°$

$\theta = 180 - \alpha, 360 - \alpha \qquad\qquad \theta = \alpha, 180 + \alpha$

$\theta = 180 - 63°26', 360 - 63°26' \qquad \theta = 45°, 180 + 45°$
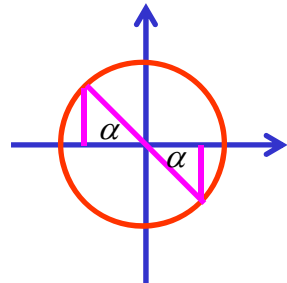
$\theta = 116°34', 296°34' \qquad\qquad\qquad \theta = 45°, 225°$

$\theta = 45°, 116°34', 225°, 296°34'$
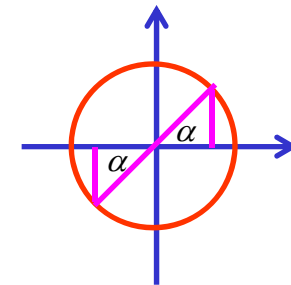
Exercise 6H; 1af, 2be, 3af, 4b, 5cd, 6c, 7a, 8a, 9d, 10adf

Old Cambridge: **Exercise 4G; 10af, 12a, 13b, 14d, 15acegi**