(6) t results

If $t = \tan \dfrac{x}{2}$

then $\sin x = \dfrac{2t}{1+t^2}$, $\cos x = \dfrac{1-t^2}{1+t^2}$

$$\dfrac{dt}{dx} = \dfrac{1}{2}\sec^2 \dfrac{x}{2}$$

$$= \dfrac{1}{2}\left(1 + \tan^2 \dfrac{x}{2}\right)$$

$$= \dfrac{1}{2}\left(1 + t^2\right)$$

$$\dfrac{dx}{dt} = \dfrac{2}{1+t^2}$$

$$dx = \dfrac{2dt}{1+t^2}$$

$$\text{e.g.} \int \frac{dx}{7 + 6\cos x}$$

$$t = \tan\frac{x}{2}$$

$$dx = \frac{2dt}{1 + t^2}$$

$$= \int \frac{\dfrac{2dt}{1 + t^2}}{7 + 6\left(\dfrac{1 - t^2}{1 + t^2}\right)}$$

$$= \int \frac{2dt}{7 + 7t^2 + 6 - 6t^2}$$

$$= \int \frac{2dt}{13 + t^2}$$

$$= \frac{2}{\sqrt{13}} \tan^{-1} \frac{t}{\sqrt{13}} + c$$

$$= \frac{2}{\sqrt{13}} \tan^{-1} \left(\frac{\tan\dfrac{x}{2}}{\sqrt{13}} \right) + c$$

If $t = \tan x$

$$\frac{dt}{dx} = \sec^2 x$$
$$= 1 + \tan^2 x$$
$$= 1 + t^2$$

$$\frac{dx}{dt} = \frac{1}{1+t^2}$$
$$dx = \frac{dt}{1+t^2}$$

In General :

If $t = \tan \dfrac{x}{a}$

$$\frac{dt}{dx} = \frac{1}{a}\sec^2 \frac{x}{a}$$
$$= \frac{1}{a}\left(1 + \tan^2 \frac{x}{a}\right)$$
$$= \frac{1}{a}\left(1 + t^2\right)$$

$$\frac{dx}{dt} = \frac{a}{1+t^2}$$
$$dx = \frac{a\,dt}{1+t^2}$$

Exercise 4G; 8bc, 15bc, 16