

(6) t results

If $t = \tan \frac{x}{2}$

then $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$

$$\begin{aligned}\frac{dt}{dx} &= \frac{1}{2} \sec^2 \frac{x}{2} \\ &= \frac{1}{2} \left(1 + \tan^2 \frac{x}{2} \right) \\ &= \frac{1}{2} (1+t^2)\end{aligned}$$

$$\frac{dx}{dt} = \frac{2}{1+t^2}$$

$dx = \frac{2dt}{1+t^2}$

$$\text{e.g. } \int \frac{dx}{7+6\cos x}$$

$$\begin{aligned}&= \int \frac{\frac{2dt}{1+t^2}}{7+6\left(\frac{1-t^2}{1+t^2}\right)} \\&= \int \frac{2dt}{7+7t^2+6-6t^2} \\&= \int \frac{2dt}{13+t^2} \\&= \frac{2}{\sqrt{13}} \tan^{-1} \frac{t}{\sqrt{13}} + c \\&= \frac{2}{\sqrt{13}} \tan^{-1} \left(\frac{\tan \frac{x}{2}}{\sqrt{13}} \right) + c\end{aligned}$$

$$t = \tan \frac{x}{2}$$

$$dx = \frac{2dt}{1+t^2}$$

If $t = \tan x$

$$\begin{aligned}\frac{dt}{dx} &= \sec^2 x \\ &= 1 + \tan^2 x \\ &= 1 + t^2\end{aligned}$$

$$\begin{aligned}\frac{dx}{dt} &= \frac{1}{1+t^2} \\ dx &= \frac{dt}{1+t^2}\end{aligned}$$

In General :

If $t = \tan \frac{x}{a}$

$$\begin{aligned}\frac{dt}{dx} &= \frac{1}{a} \sec^2 \frac{x}{a} \\ &= \frac{1}{a} \left(1 + \tan^2 \frac{x}{a} \right) \\ &= \frac{1}{a} (1 + t^2)\end{aligned}$$

$$\frac{dx}{dt} = \frac{a}{1+t^2}$$

$$dx = \frac{adt}{1+t^2}$$

Exercise 4G; 8bc, 15bc, 16