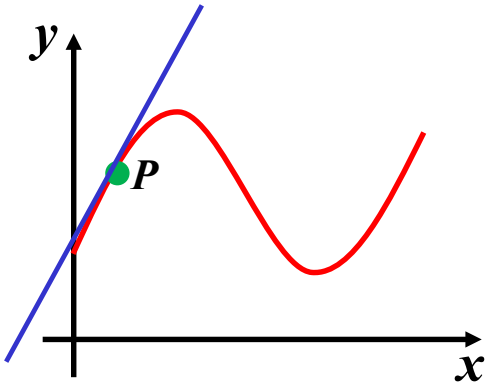
Rates of Change

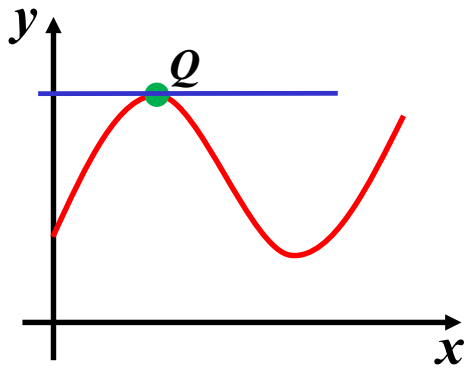
A derivative measures the rate of one quantity changing with respect to another quantity

The steeper the curve, the faster the quantity changes



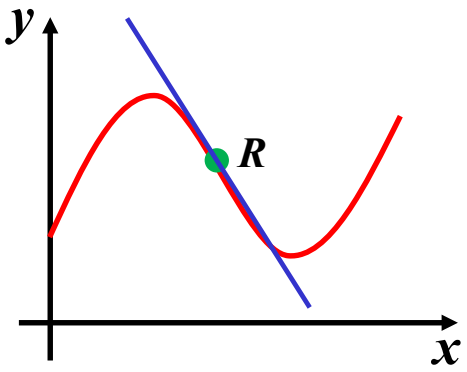
at P, $\dfrac{dy}{dx} > 0$

$f(x)$ is ***increasing***



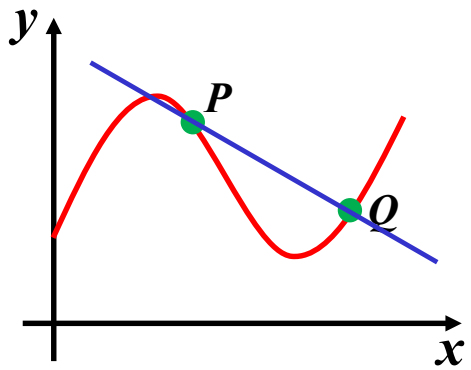
at Q, $\dfrac{dy}{dx} = 0$

$f(x)$ is ***stationary***

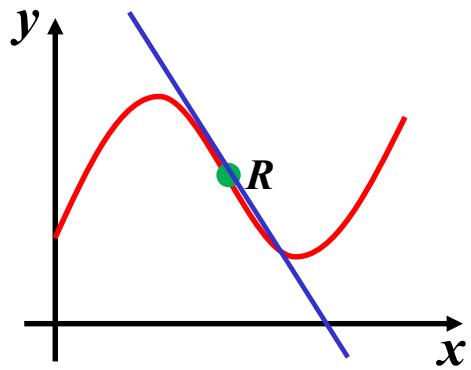


at R, $\dfrac{dy}{dx} < 0$

$f(x)$ is ***decreasing***



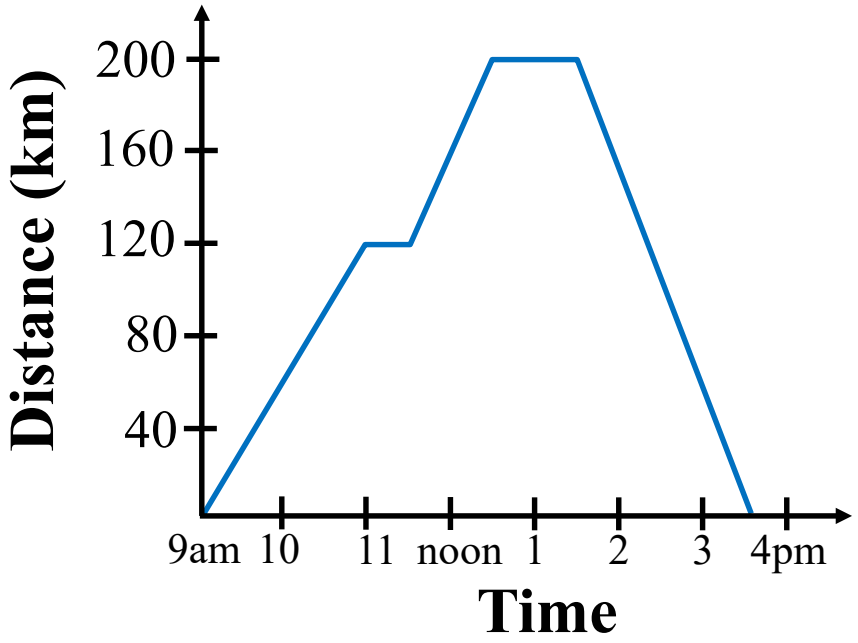
the slope of the secant PQ gives the **average** rate of change



the slope of the tangent at R gives the **instantaneous** rate of change

Travel Graphs

e.g. A coach leaves the depot at 9 am for a one day tour



(i) How long was the first stop?

30 minutes

(ii) When did the coach start to return to the depot?

1:30 pm

(iii) How fast was the coach travelling on the return trip?

$s = \frac{d}{t}$

$= \frac{200}{2}$

$= 100$ km/h

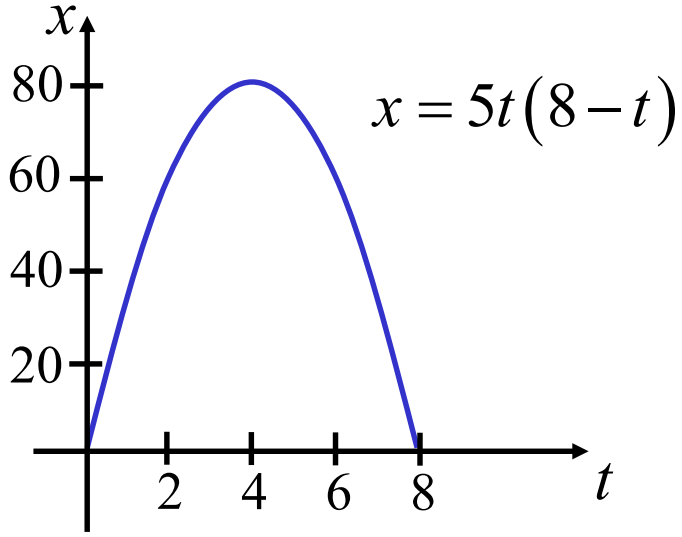
(iv) Calculate the average speed for the whole trip?

$s = \frac{d}{t}$

$= \frac{400}{6.5}$

$= 61.5$ km/h

(ii) A ball is bounced and its distance from the ground is graphed.



Distance = total amount travelled

Displacement = how far from the starting point

(i) Find the height of the ball after 1 second

when $t = 1, x = 5(1)(8-1)$

$= 35$

After 1 second the ball is 35 metres above the ground

(ii) At what other time is the ball this same height above the ground?

when $x = 35$, $\quad 5t(8-t) = 35$

$$t(8-t) = 7$$

$$8t - t^2 = 7$$

$$t^2 - 8t + 7 = 0$$

$$(t-1)(t-7) = 0$$

$$t = 1 \quad \text{or} \quad t = 7$$

$\therefore$ ball is 35 metres above ground again after 7 seconds

Average velocity $= \dfrac{\text{change in displacement}}{\text{change in time}}$

$$= \frac{x_2 - x_1}{t_2 - t_1}$$

(iii) Find the average velocity during the 1st second

$$\text{average velocity} = \frac{x_2 - x_1}{t_2 - t_1}$$
$$= \frac{35 - 0}{1 - 0}$$
$$= 35$$

$\therefore$ average velocity during the 1st second was 35m/s

(iv) Find the average velocity during the fifth second

when $t = 4, x = 5(4)(8-4)$
$=80$

when $t = 5, x = 5(5)(8-5)$
$=75$

$$\text{average velocity} = \frac{x_2 - x_1}{t_2 - t_1}$$
$$= \frac{75 - 80}{5 - 4}$$
$$= -5$$

$\therefore$ average velocity during the 5th second was -5m/s

(iv) Find the average velocity during its 8 seconds in the air

$$\text{average velocity} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{0-0}{8-0}$$

$$= 0$$

$\therefore$ average velocity during the 8 seconds was 0m/s

Average speed = $\dfrac{\text{distance travelled}}{\text{time taken}}$

(v) Find the average speed during its 8 seconds in the air

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{160}{8}$$

$$= 20$$

$\therefore$ average speed during the 8 seconds was 20m/s

Exercise 9A; 3, 5, 6, 7, 8, 9, 10, 12, 13