

Expectation

Let X be a **discrete random variable**, then the expected value of X is;

$$E(X) = \sum xp(x)$$

where $p(x) = P(X = x) \geq 0$

Note: $E(X) = \mu$ (arithmetic mean)

$E(X)$ is a measure of central tendency

Laws of Expectation

1. If a and b are constants; $E(aX + b) = aE(X) + b$

Proof:

$$\begin{aligned} E(aX + b) &= \sum (ax + b) p(x) \\ &= \sum axp(x) + \sum bp(x) \\ &= a \sum xp(x) + b \sum p(x) \\ &= \underline{aE(X) + b} \end{aligned}$$

2. $E(X + Y) = E(X) + E(Y)$

e.g. In a marketing survey, 25 families are polled to find the number of litres of milk consumed during a particular week.

The results were;

# of litres	0	1	2	3	4	5
# of families	2	5	9	5	3	1

Based on this data, how many litres of milk would you expect a similar family to consume in a week?

x	0	1	2	3	4	5	Σ
$p(x)$	$\frac{2}{25}$	$\frac{5}{25}$	$\frac{9}{25}$	$\frac{5}{25}$	$\frac{3}{25}$	$\frac{1}{25}$	1
$xp(x)$	0	$\frac{5}{25}$	$\frac{18}{25}$	$\frac{15}{25}$	$\frac{12}{25}$	$\frac{5}{25}$	2.2

$$E(X) = \sum xp(x) \quad \text{or} \quad E(X) = \mu$$

$$= 2.2$$

$$= \frac{0 \times 2 + 1 \times 5 + 2 \times 9 + 3 \times 5 + 4 \times 3 + 5 \times 1}{25}$$

$$= 2.2$$

We would expect a family to consume 2.2 L of milk weekly

*Note: Random variables have an expected value
Sample spaces have a mean*

Exercise 13B; 2, 3, 4a, 5, 6, 7adf, 8, 9, 10, 11, 12, 13, 14