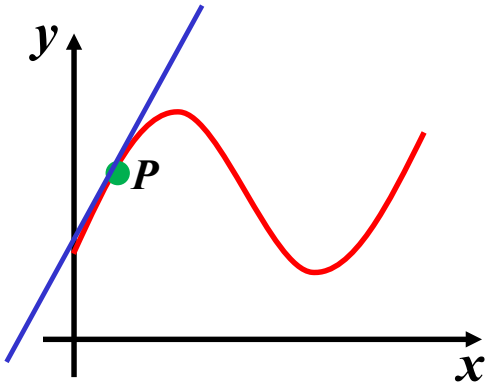
Concavity & Rates of Change

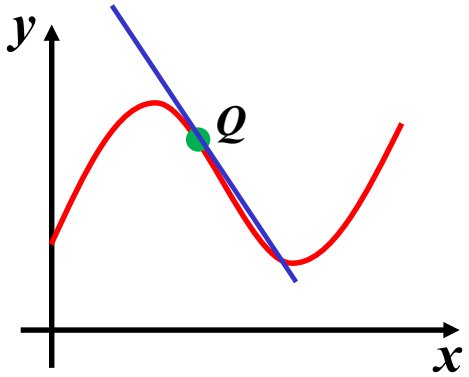
The second derivative measures how the first derivative changes

The greater the magnitude of the concavity the faster the rate of change changes



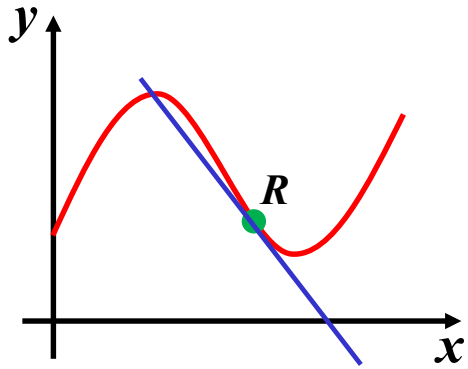
at P, $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$

$f(x)$ is ***increasing*** at a ***decreasing*** *rate*



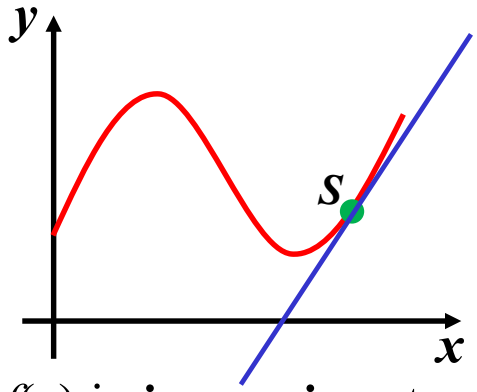
at Q, $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} < 0$

$f(x)$ is ***decreasing*** at an ***increasing*** *rate*



at R, $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} > 0$

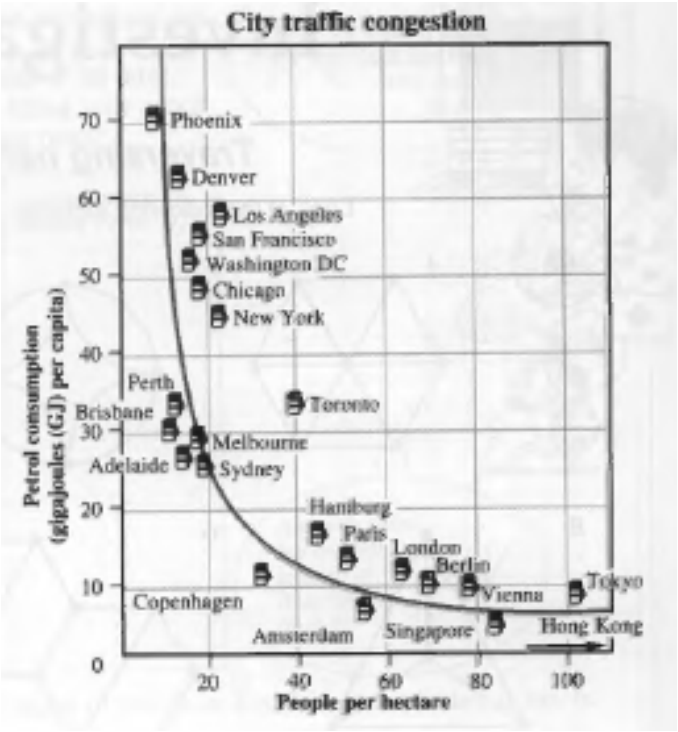
$f(x)$ is ***decreasing*** at a ***decreasing*** *rate*



at S, $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$

$f(x)$ is ***increasing*** at a ***increasing*** *rate*

e.g. (i) This graph shows petrol consumption versus population density in 22 cities.



At less than 20 people per hectare, people depend more on their cars for transport and there is little public transport.

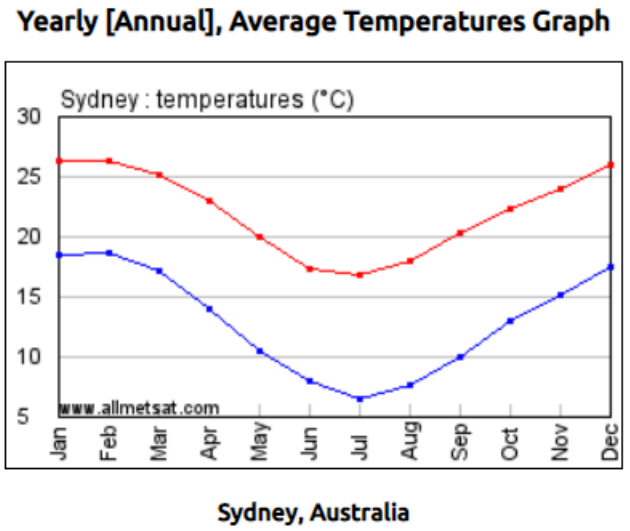
What can be concluded about petrol consumption as the population density increases?

Petrol consumption per capita decreases at a decreasing rate

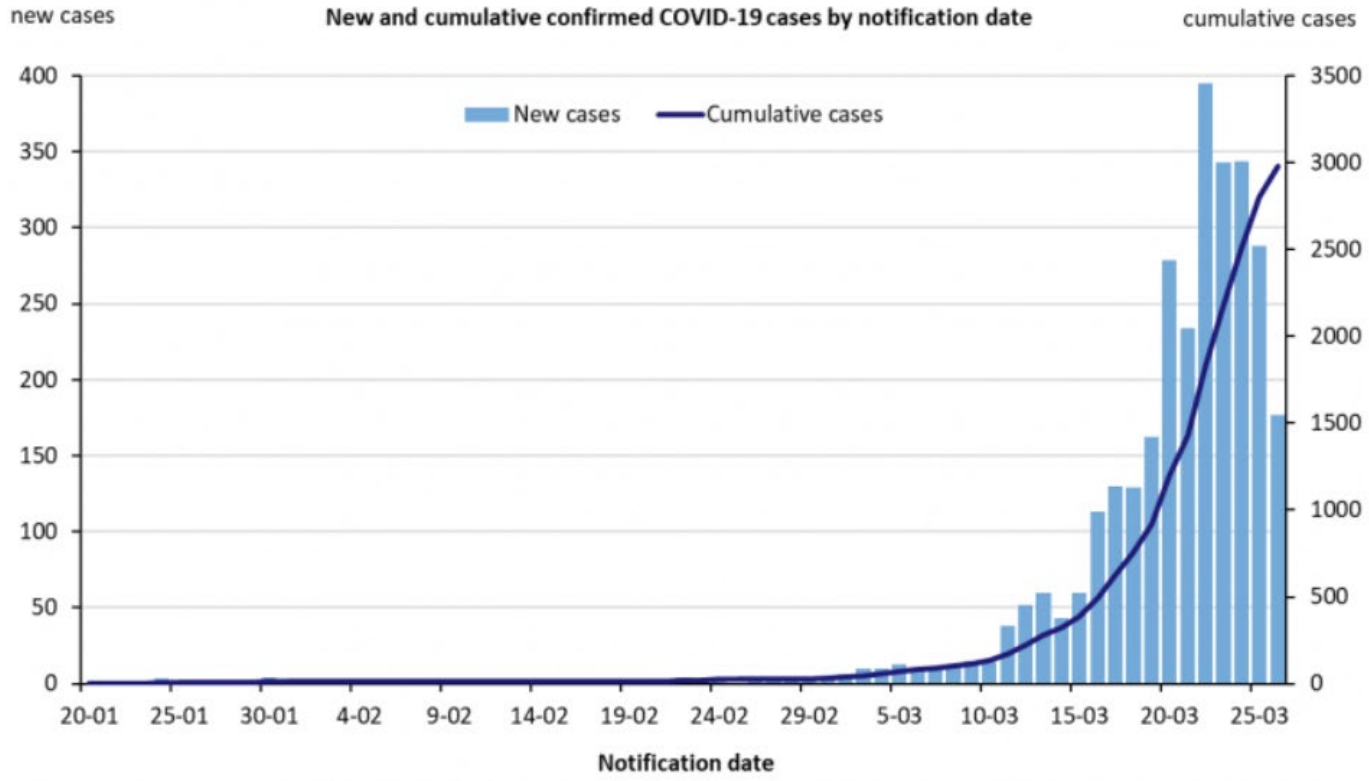
(ii) How does the temperature change during Autumn?

At the start of Autumn temperature is decreasing at an increasing rate.

However during Autumn that rate slows down and by the end of Autumn the temperature is still decreasing, but now at a decreasing rate



(iii) On March 22, the Prime Minister announced Stage 1 shutdown restrictions in order to slow down the rate of new cases.



Given that it takes about 5 to 6 days for symptoms to show, do you think the measure has been effective?

Five days later, 27/3, the cumulative cases curve appears to have inflected and changed to concave down.

This would suggest that whilst the number of cases are still increasing, they are now growing at a decreasing rate i.e the rate is slowing down

Whilst is still very early, yes, it does appear that the measures are effective

Exercise 9D; 2, 4, 6, 7, 8, 10, 12, 13, 14