Rules For Differentiation

(1) $y = c$

$$f(x) = c$$

$$f(x+h) = c$$

$$f'(x) = \lim_{h \to 0} \frac{c - c}{h}$$

$$= \lim_{h \to 0} \frac{0}{h}$$

$$= \lim_{h \to 0} 0$$

$$= 0$$

(2) $y = kx$

$$f(x) = kx$$

$$f(x+h) = k(x+h)$$

$$= kx + kh$$

$$f'(x) = \lim_{h \to 0} \frac{kx + kh - kx}{h}$$

$$= \lim_{h \to 0} \frac{kh}{h}$$

$$= \lim_{h \to 0} k$$

$$= k$$

(3) $y = x^n$

$$a^2 - b^2 = (a-b)(a+b)$$
$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$
$$a^4 - b^4 = (a-b)(a^3+a^2b+ab^2+b^3)$$
$$\vdots$$
$$a^n - b^n = (a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\ldots+a^2b^{n-3}+ab^{n-2}+b^{n-1})$$

$$f(x) = x^n \qquad f(x+h) = (x+h)^n$$

$$f'(x) = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h}$$
$$= \lim_{h\to 0}\frac{(x+h-x)\{(x+h)^{n-1}+(x+h)^{n-2}x+\ldots+(x+h)x^{n-2}+x^{n-1}\}}{h}$$
$$= \lim_{h\to 0}\frac{h\{(x+h)^{n-1}+(x+h)^{n-2}x+\ldots+(x+h)x^{n-2}+x^{n-1}\}}{h}$$
$$= \lim_{h\to 0}(x+h)^{n-1}+(x+h)^{n-2}x+\ldots+(x+h)x^{n-2}+x^{n-1}$$

$$f'(x)=\lim_{h\to 0}(x+h)^{n-1}+(x+h)^{n-2}x+\ldots+(x+h)x^{n-2}+x^{n-1}$$

$$=x^{n-1}+x^{n-1}+\ldots+x^{n-1}+x^{n-1}$$

$$=nx^{n-1}$$

e.g. $(i)\quad y=7$

$$\frac{dy}{dx}=0$$

$(ii)\quad y=37x$

$$\frac{dy}{dx}=37$$

$(iii)\quad y=x^{10}$

$$\frac{dy}{dx}=10x^9$$

$(iv)\quad y=3x^2+6x+2$

$$\frac{dy}{dx}=6x+6$$

$(v)\quad y=(2x+1)^2$

$$=4x^2+4x+1$$

$$\frac{dy}{dx}=8x+4$$

(vi) If $f(x)=x^3-3$, find $f'(2)$

$$f(x)=x^3-3$$

$$f'(x)=3x^2$$

$$f'(2)=3(2)^2$$

$$=12$$

(vii) Find the equation of the tangent to the curve $y = 5x^3 - 6x^2 + 2$ at the point $(1,1)$

$$y = 5x^3 - 6x^2 + 2$$

$$\frac{dy}{dx} = 15x^2 - 12x$$

when $x = 1, \frac{dy}{dx} = 15(1)^2 - 12(1)$

$$= 3$$

$\therefore$ required slope $= 3$

$$y - 1 = 3(x - 1)$$

$$y - 1 = 3x - 3$$

$$\underline{3x - y - 2 = 0}$$

$(viii)$ Find the points on the curve $y = x^3 - 12x$ where the tangents are horizontal

$$y = x^3 - 12x$$

$$\frac{dy}{dx} = 3x^2 - 12$$

tangents are horizontal when $\frac{dy}{dx} = 0$

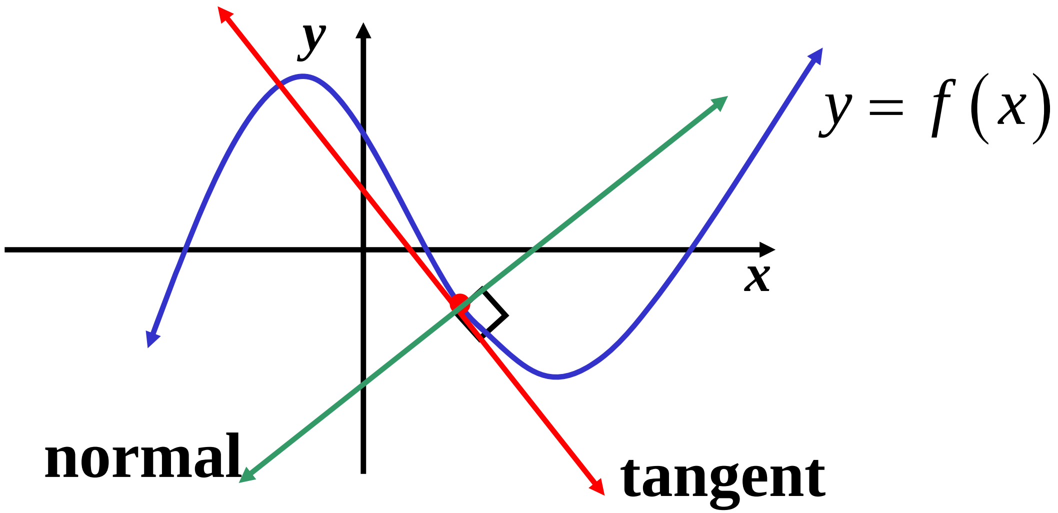
i.e. $3x^2 - 12 = 0$

$$x^2 = 4$$

$$x = \pm 2$$

$\therefore$ tangents are horizontal at $(-2,16)$ and $(2,-16)$

A **normal** is a line perpendicular to the tangent at the point of contact



(ix) Find the equation of the normal to the curve $y = 4x^2 - 3x + 2$ at the point $(3, 29)$

$$y = 4x^2 - 3x + 2$$

$$\frac{dy}{dx} = 8x - 3$$

when $x = 3, \frac{dy}{dx} = 8(3) - 3$

$$= 21$$

$\therefore$ required slope $= -\frac{1}{21}$

$$y - 29 = -\frac{1}{21}(x - 3)$$

$$21y - 609 = -x + 3$$

$$x + 21y - 612 = 0$$

A note on setting out

When differentiating, you are actually solving an equation i.e. "**what you do to one side of the equation you do to the other**"

$$y = x^2 + 2x$$

$$\frac{d(y)}{dx} = \frac{d\left(x^2 + 2x\right)}{dx}$$

$$\frac{d(y)}{dy} \times \frac{dy}{dx} = \frac{d\left(x^2 + 2x\right)}{dx}$$

$$\frac{dy}{dx} = 2x + 2$$

"differentiate both sides with respect to *x*"

Exercise 9C; 1ace etc, 2ace etc , 3ace etc, 4b, 5b, 6bdf, 9, 10, 12, 16ace

Exercise 9D; 5a, 7, 8, 9b, 11a, 12, 14bdf, 15, 17, 22

Match the notation being used

(1) $y = x^2 + 2x$ ***OR*** $y = x^2 + 2x$

$\frac{dy}{dx} = 2x + 2$ $\qquad$ $y' = 2x + 2$

(2) $f(x) = x^2 + 2x$

$f'(x) = 2x + 2$

(3) $\frac{d\left(x^2 + 2x\right)}{dx} = 2x + 2$