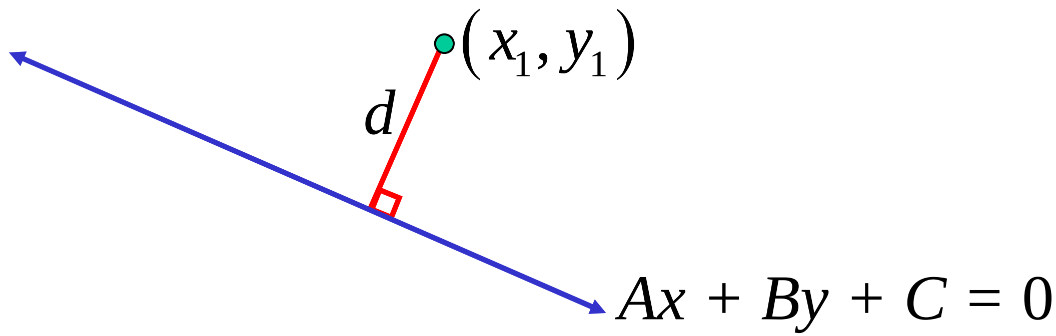
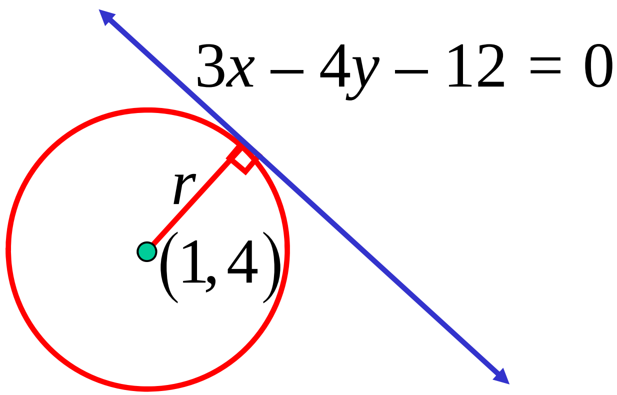
Perpendicular Distance Formula

The shortest distance from a point to a line is the **perpendicular distance.**



$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

e.g. Find the equation of the circle with tangent $3x - 4y - 12 = 0$ and centre (1,4).



$$r = \frac{|3(1) - 4(4) - 12|}{\sqrt{3^2 + (-4)^2}}$$

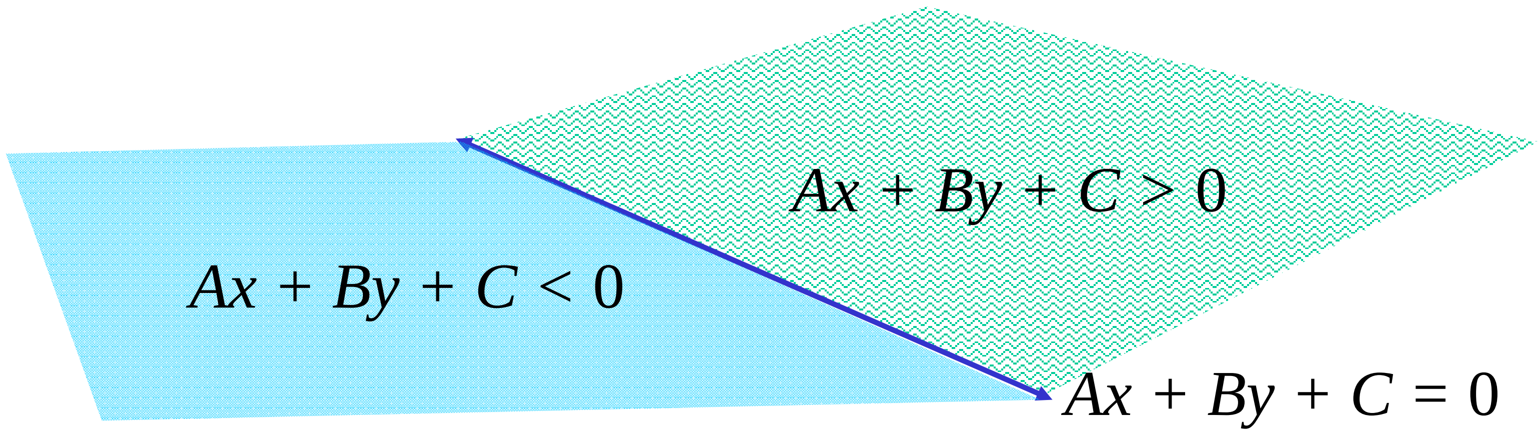
$$= \frac{25}{\sqrt{25}}$$

$$= 5 \text{ units}$$

$\therefore$ the circle is

$$(x-1)^2 + (y-4)^2 = 25$$

If $\left(Ax_1 + By_1 + C\right)$ has different signs for different points, they are on different sides of the line.



Equation of a line through a point and intersection of another two lines

e.g. Find the equation of the line that passes through the intersection of $2x + y + 1 = 0$ and $3x + 5y - 9 = 0$ and the point (1,2).

$$\begin{aligned} 2x + y + 1 &= 0 \\ 3x + 5y - 9 &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 10x + 5y &= -5 \quad (-) \\ 3x + 5y &= 9 \\ \hline 7x \qquad &= -14 \\ x &= -2 \end{aligned}$$

$$\therefore 2(-2) + y + 1 = 0$$

$$y = 3$$

$\therefore$ the lines intersect at $(-2,3)$

$$m = \frac{3-2}{-2-1} = \frac{1}{-3}$$

$$y - 2 = -\frac{1}{3}(x-1)$$

$$3y - 6 = -x + 1$$

$$\underline{x + 3y - 7 = 0}$$

Alternatively

point of intersection lies on line 1

$\therefore a_1x + b_1y + c_1 = 0$

point of intersection lies on line 2

$\therefore a_2x + b_2y + c_2 = 0$

$$a_1x + b_1y + c_1 + k\left(a_2x + b_2y + c_2\right) = 0$$

$$2x + y + 1 + k\left(3x + 5y - 9\right) = 0$$

$$(1,2): 2(1) + (2) + 1 + k\left(3(1) + 5(2) - 9\right) = 0$$

$$5 + 4k = 0$$

$$4k = -5$$

$$k = -\frac{5}{4}$$

$$2x + y + 1 - \frac{5}{4}(3x + 5y - 9) = 0$$

$$8x + 4y + 4 - 15x - 25y + 45 = 0$$

$$7x + 21y - 49 = 0$$

$$x + 3y - 7 = 0$$

Exercise 7E; 1, 3, 4, 8, 9, 10

Past HSC Questions