Bernoulli Distribution

A **Bernoulli Distribution** is the simplest type of discrete probability distribution.

Bernoulli random variables have two possible values;

1 (success) or 0 (failure)

Let X represent a Bernoulli random variable, (Bernoulli trial), with a probability density function

$$P(X = x) = \begin{cases} p & , x = 1 \\ 1 - p & , x = 0 \end{cases}$$

which can also be expressed as;

$$P(X = x) = (1 - p)^{1 - x} p^{x}$$

x	**0**	**1**	**Σ**
$p(x)$	$1 - p$	p	1
$xp(x)$	0	p	p
$x^2p(x)$	0	p	p

$$
\begin{aligned}
E(X) &= \Sigma xp(x) \\
&= p
\end{aligned}
$$

$$
\begin{aligned}
Var(X) &= E(X^2) - \mu^2 \\
&= p - p^2 \\
&= p(1 - p)
\end{aligned}
$$

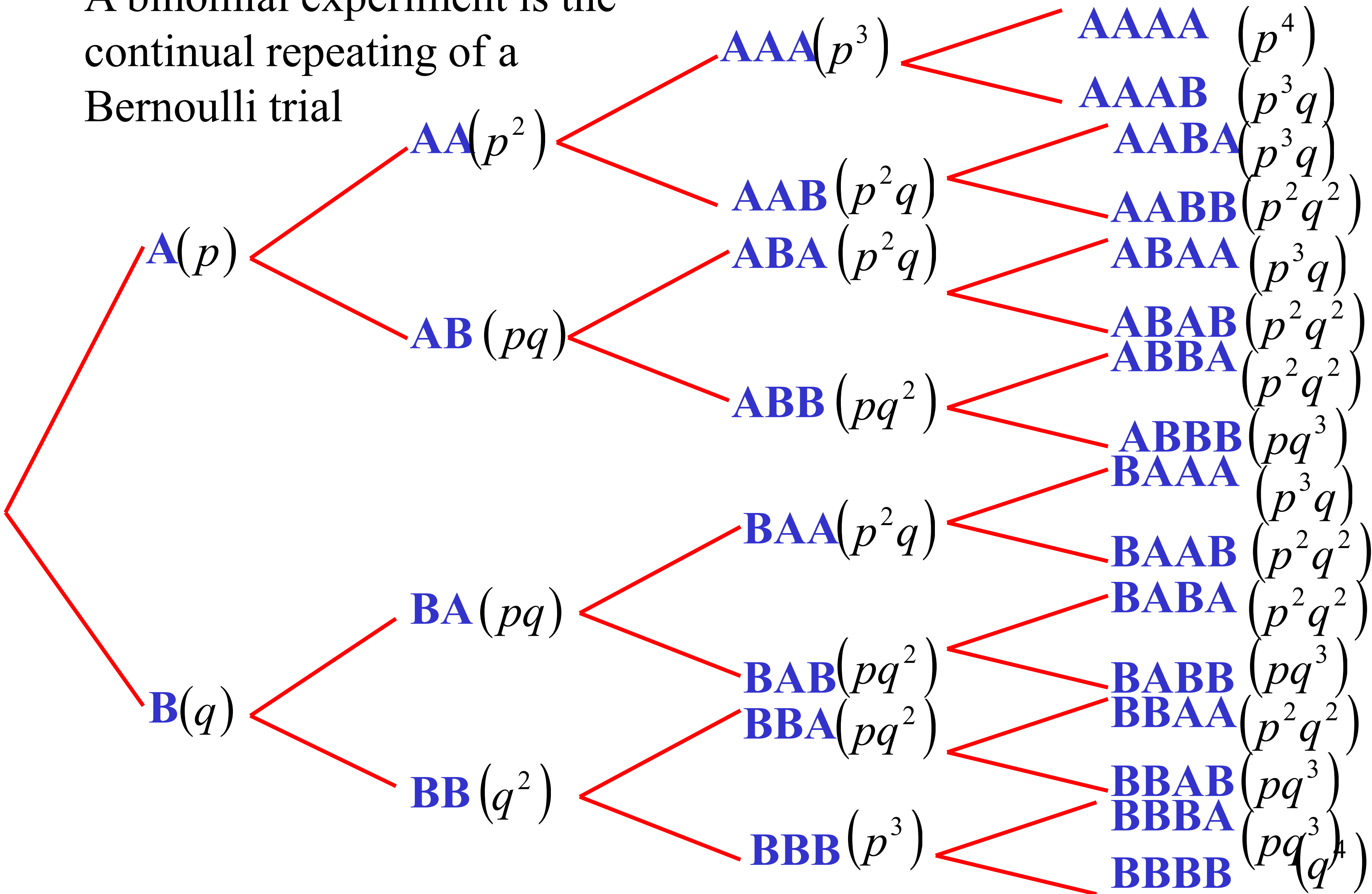
If $X \sim \text{Bern}(p)$

$$E(X) = p$$

$$Var(X) = p(1 - p)$$

Binomial Probability

A binomial experiment is the continual repeating of a Bernoulli trial



1 Event

$P(A) = p$

$P(B) = q$

2 Events

$P(AA) = p^2$

$P(AandB) = 2pq$

$P(BB) = q^2$

3 Events

$P(AAA) = p^3$

$P(2AandB) = 3p^2q$

$P(Aand2B) = 3pq^2$

$P(BBB) = q^3$

4 Events

$P(AAAA) = p^4$

$P(3AandB) = 4p^3q$

$P(2Aand2B) = 6p^2q^2$

$P(Aand3B) = 4pq^3$

$P(BBBB) = q^4$

If X represents a Bernoulli trial, then the probability that X occurs exactly k times in n trials is;

$$P(X = k) = {}^nC_k q^{n-k} p^k$$

$Note: q = 1 - p$

e.g.(i) A bag contains 30 black balls and 20 white balls.
Seven drawings are made (with replacement), what is the probability of drawing; Let X be the number of black balls drawn

a) All black balls?

$$P(X=7) = {}^7C_7\left(\frac{2}{5}\right)^0\left(\frac{3}{5}\right)^7$$

$$= \frac{{}^7C_7 3^7}{5^7}$$

$$= \frac{2187}{78125}$$

b) 4 black balls?

$$P(X=4) = {}^7C_4\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^4$$

$$= \frac{{}^7C_4 2^3 3^4}{5^7}$$

$$= \frac{4536}{15625}$$

(ii) At an election 30% of voters favoured Party A.
If at random an interviewer selects 5 voters, what is the probability that;

a) 3 favoured Party A?

Let X be the number favouring Party A

$$P(X = 3) = {}^5C_3\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^3$$

$$= \frac{{}^5C_3 7^2 3^3}{10^5} = \frac{1323}{10000}$$

b) majority favour A?

$$P(X \geq 3) = {}^5C_3\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^3 + {}^5C_4\left(\frac{7}{10}\right)^1\left(\frac{3}{10}\right)^4 + {}^5C_5\left(\frac{7}{10}\right)^0\left(\frac{3}{10}\right)^5$$

$$= \frac{{}^5C_3 7^2 3^3 + {}^5C_4 7 \cdot 3^4 + {}^5C_5 3^5}{10^5}$$

$$= \frac{4077}{25000}$$

c) at most 2 favoured A?

$$P(X \leq 2) = 1 - P(X \geq 3)$$

$$= 1 - \frac{4077}{25000}$$

$$= \frac{20923}{25000}$$

2005 Extension 1 HSC Q6a)

There are five matches on each weekend of a football season. Megan takes part in a competition in which she earns 1 point if she picks more than half of the winning teams for a weekend, and zero points otherwise.

The probability that Megan correctly picks the team that wins in any given match is $\frac{2}{3}$

(i) Show that the probability that Megan earns one point for a given weekend is $0 \cdot 7901$, correct to four decimal places.

Let X be the number of matches picked correctly

$$P(X \geq 3) = {}^5C_3\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3 + {}^5C_4\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^4 + {}^5C_5\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^5$$

$$= \frac{{}^5C_3 2^3 + {}^5C_4 2^4 + {}^5C_5 2^5}{3^5}$$

$$= 0 \cdot 7901$$

(ii) Hence find the probability that Megan earns one point every week of the eighteen week season. Give your answer correct to two decimal places.

Let Y be the number of weeks Megan earns a point

$$P(Y=18) = {}^{18}C_{18}(0\cdot 2099)^{0}(0\cdot 7901)^{18}$$
$$= 0\cdot 01 \text{ (to 2 dp)}$$

(iii) Find the probability that Megan earns at most 16 points during the eighteen week season. Give your answer correct to two decimal places.

$$P(Y \leq 16) = 1 - P(Y \geq 17)$$
$$= 1 - {}^{18}C_{17}(0\cdot 2099)^{1}(0\cdot 7901)^{17} - {}^{18}C_{18}(0\cdot 2099)^{0}(0\cdot 7901)^{18}$$
$$= 0\cdot 92 \text{ (to 2 dp)}$$

2007 Extension 1 HSC Q4a)

In a large city, 10% of the population has green eyes.

(i) What is the probability that two randomly chosen people have green eyes?

$$P(2\text{ green}) = 0.1 \times 0.1$$
$$= 0.01$$

(ii) What is the probability that exactly two of a group of 20 randomly chosen people have green eyes? Give your answer correct to three decimal eyes.

Let X be the number of people with green eyes

$$P(X = 2) = {}^{20}C_2 (0.9)^{18} (0.1)^2$$
$$= 0.2851\ldots$$
$$= 0.285 \quad \text{(to 3 dp)}$$

(iii) What is the probability that more than two of a group of 20 randomly chosen people have green eyes? Give your answer correct to two decimal places.

$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - {}^{20}C_2 (0\cdot 9)^{18} (0\cdot 1)^2 - {}^{20}C_1 (0\cdot 9)^{19} (0\cdot 1)^1 - {}^{20}C_0 (0\cdot 9)^{20} (0\cdot 1)^0$$

$$= 0.3230\ldots$$

$$= 0\cdot 32 \quad \text{(to 2 dp)}$$

Exercise 17A;
2, 4, 7, 8, 11, 13, 15, 17, 19, 20, 21, 22