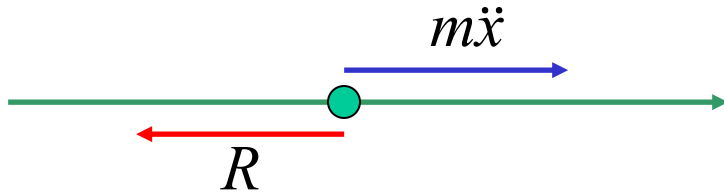


# *Resisted Motion*

Resistance is ALWAYS in the OPPOSITE direction to the motion.

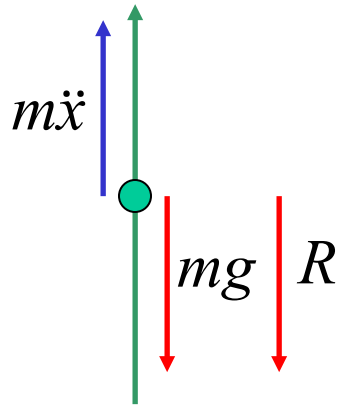
*(Newton's 3<sup>rd</sup> Law)*

Case 1 (horizontal line)



$$m\ddot{x} = -R$$
$$\ddot{x} = -\frac{R}{m}$$

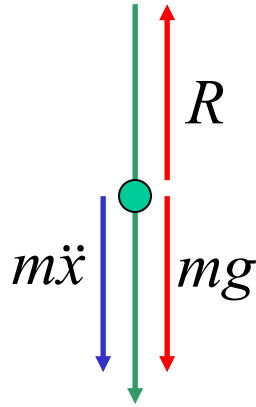
Case 2 (upwards motion)



$$m\ddot{x} = -mg - R$$
$$\ddot{x} = -g - \frac{R}{m}$$

NOTE: greatest height still occurs when  $v = 0$

### Case 3 (downwards motion)



$$m\ddot{x} = mg - R$$

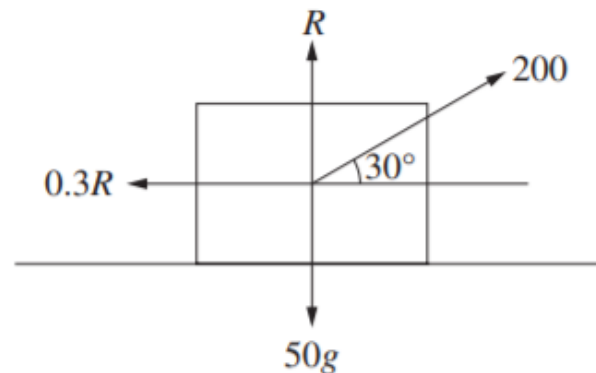
$$\ddot{x} = g - \frac{R}{m}$$

NOTE: terminal velocity occurs  
when  $\ddot{x} = 0$

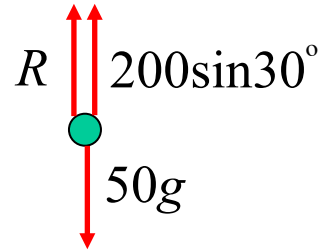
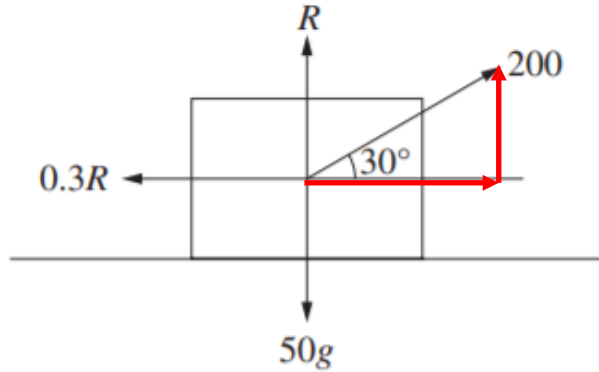
e.g. (i) 2020 Extension 2 HSC Question 12 a)

A 50 kilogram box is initially at rest. The box is pulled along the ground with a force of 200 newtons at an angle of  $30^\circ$  to the horizontal. The box experiences a resistive force of  $0.3R$  newtons, where  $R$  is the normal force, as shown in the diagram.

Take the acceleration  $g$  due to gravity to be  $10 \text{ m/s}^2$ .



a) By resolving the forces vertically, show that  $R = 400$



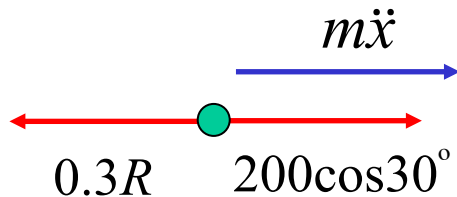
vertical  $F = 0$

$$R + 200 \sin 30^\circ - 50g = 0$$

$$R + 100 - 500 = 0$$

$$\underline{R = 400}$$

b) Show that the net force horizontally is approximately 53.2 newtons.



net force is equivalent to  
resultant force

$$m\ddot{x} = 200 \cos 30^\circ - 0.3R$$

$$F_H = 100\sqrt{3} - 120$$

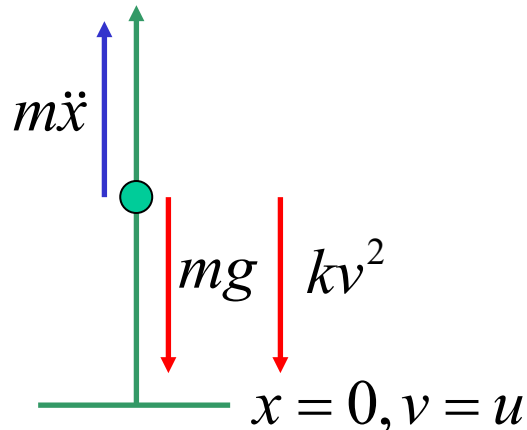
$$= \underline{53.2 \text{ newtons (to 1 dp)}}$$

c) Find the velocity of the box after the first three seconds.

$$\begin{aligned}
 m\ddot{x} &= 53.2 \\
 \frac{dv}{dt} &= \frac{53.2}{50} \\
 &= \frac{133}{125}
 \end{aligned}
 \qquad
 \begin{aligned}
 \int_0^v dv &= \frac{133}{125} \int_0^3 dt \\
 v &= \frac{133}{125}(3) \\
 &= \underline{3.2 \text{ m/s (to 1 dp)}}
 \end{aligned}$$

(ii) A particle is projected vertically upwards with a velocity of  $u$  m/s in a resisting medium.

Assuming that the retardation due to this resistance is equal to  $kv^2$  find expressions for the greatest height reached and the time taken to reach that height.



$$m\ddot{x} = -mg - kv^2$$

$$\ddot{x} = -g - \frac{k}{m}v^2$$

$$v \frac{dv}{dx} = \frac{-mg - kv^2}{m}$$

$$\frac{dx}{dv} = \frac{mv}{-mg - kv^2}$$

$$\int_0^x dx = -m \int_u^0 \frac{v dv}{mg + kv^2}$$

$$x = \frac{m}{2k} \int_0^u \frac{2kv dv}{mg + kv^2}$$

$$= \frac{m}{2k} \left[ \log(mg + kv^2) \right]_0^u$$

$$= \frac{m}{2k} \left\{ \log(mg + ku^2) - \log(mg) \right\}$$

$$= \frac{m}{2k} \log \left( \frac{mg + ku^2}{mg} \right)$$

$$= \frac{m}{2k} \log \left( 1 + \frac{ku^2}{mg} \right)$$

$\therefore$  the greatest height

is  $\frac{m}{2k} \log \left( 1 + \frac{ku^2}{mg} \right)$  metres

$$\ddot{x} = -g - \frac{k}{m}v^2$$

$$\frac{dv}{dt} = \frac{-mg - kv^2}{m}$$

$$\int_0^t dt = -m \int_u^0 \frac{dv}{mg + kv^2}$$

$$t = \frac{m}{k} \int_0^u \frac{dv}{\frac{mg}{k} + v^2}$$

$$= \frac{m}{k} \left[ \sqrt{\frac{k}{mg}} \tan^{-1} \left( \sqrt{\frac{k}{mg}} v \right) \right]_0^u$$

$$= \sqrt{\frac{m}{kg}} \left\{ \tan^{-1} \left( \sqrt{\frac{k}{mg}} u \right) - \tan^{-1} 0 \right\} \therefore \text{it takes } \sqrt{\frac{m}{kg}} \tan^{-1} \left( \sqrt{\frac{k}{mg}} u \right) \text{ seconds}$$

$$= \sqrt{\frac{m}{kg}} \tan^{-1} \left( \sqrt{\frac{k}{mg}} u \right)$$

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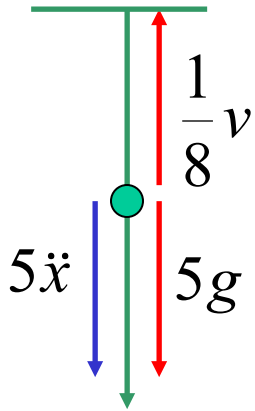
to reach the greatest height.

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(iii) A body of mass 5kg is dropped from a height at which the gravitational acceleration is  $g$ .

Assuming that air resistance is proportional to speed  $v$ , the constant of proportion being  $\frac{1}{8}$ , find;

a) the velocity after time  $t$ .



$$5\ddot{x} = 5g - \frac{1}{8}v$$

$$\ddot{x} = g - \frac{1}{40}v$$

$$\frac{dv}{dt} = \frac{40g - v}{40}$$

$$\int_0^t dt = 40 \int_0^v \frac{dv}{40g - v}$$

$$t = -40 \left[ \log(40g - v) \right]_0^v$$

$$= -40 \{ \log(40g - v) - \log(40g) \}$$

$$= 40 \log \left( \frac{40g}{40g - v} \right)$$

$$\frac{t}{40} = \log\left(\frac{40g}{40g - v}\right)$$

$$\frac{40g}{40g - v} = e^{\frac{t}{40}}$$

$$\frac{40g - v}{40g} = e^{-\frac{t}{40}}$$

$$40g - v = 40ge^{-\frac{t}{40}}$$

$$v = 40g - 40ge^{-\frac{t}{40}}$$

$$v = 40g\left(1 - e^{-\frac{t}{40}}\right)$$

---

b) the terminal velocity

terminal velocity occurs when  $\ddot{x} = 0$

$$\text{i.e. } 0 = g - \frac{1}{40}v$$

$$v = 40g$$

OR

$$\begin{aligned}\lim_{t \rightarrow \infty} v &= \lim_{t \rightarrow \infty} 40g\left(1 - e^{-\frac{t}{40}}\right) \\ &= 40g\end{aligned}$$

$\therefore$  terminal velocity is  $40g$  m/s



c) The distance it has fallen after time  $t$

$$\frac{dx}{dt} = 40g \left( 1 - e^{-\frac{t}{40}} \right)$$

$$\int_0^x dx = 40g \int_0^t \left( 1 - e^{-\frac{t}{40}} \right) dt$$

$$x = 40g \left[ t + 40e^{-\frac{t}{40}} \right]_0^t$$

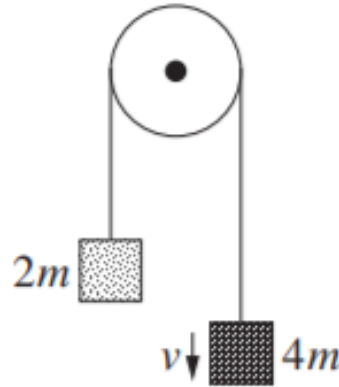
$$x = 40g \left\{ t + 40e^{-\frac{t}{40}} - 0 - 40 \right\}$$

$$\underline{x = 40gt + 1600ge^{-\frac{t}{40}} - 1600g}$$

(iv) 2020 Extension 2 HSC Question 16 a)

Two masses,  $2m$  kg and  $4m$  kg, are attached by a light string. The string is placed over a smooth pulley as shown.

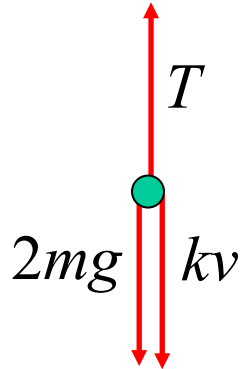
The two masses are at rest before being released and  $v$  is the velocity of the larger mass at time  $t$  seconds after they are released.



The force due to air resistance on each mass has magnitude  $k v$ , where  $k$  is a positive constant.

a) Show that  $\frac{dv}{dt} = \frac{gm - kv}{3m}$

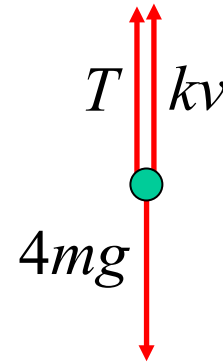
**Forces on left mass**



$$2m\ddot{x} = T - 2mg - kv$$

$$T = 2m\ddot{x} + 2mg + kv$$

**Forces on right mass**



$$4m\ddot{x} = 4mg - T - kv$$

$$= 4mg - (2m\ddot{x} + 2mg + kv) - kv$$

$$6m\ddot{x} = 2mg - 2kv$$

$$\ddot{x} = \frac{mg - kv}{3m}$$

$$\frac{dv}{dt} = \frac{gm - kv}{3m}$$

---

b) Given that  $v < \frac{gm}{k}$ , show that when  $t = \frac{3m}{2} \ln 2$  the velocity of the larger mass is  $\frac{gm}{2k}$

$$\frac{dv}{dt} = \frac{mg - kv}{3m}$$

$$v \quad \frac{3m}{k} \ln 2$$

$$\int_0^v \frac{3m \, dv}{mg - kv} = \int_0^{\frac{3m}{k} \ln 2} dt$$

$$-\frac{3m}{k} \int_0^v \frac{-k \, dv}{mg - kv} = \frac{3m}{k} \ln 2$$

$$[\ln |mg - kv|]_v^0 = \ln 2$$

$$\ln \left| \frac{mg}{mg - kv} \right| = \ln 2$$

$$\text{as } v < \frac{gm}{k} \Rightarrow \frac{mg}{mg - kv} > 0$$

$$\frac{mg}{mg - kv} = 2$$

$$mg = 2mg - 2kv$$

$$2kv = mg$$

$$v = \frac{mg}{2k}$$

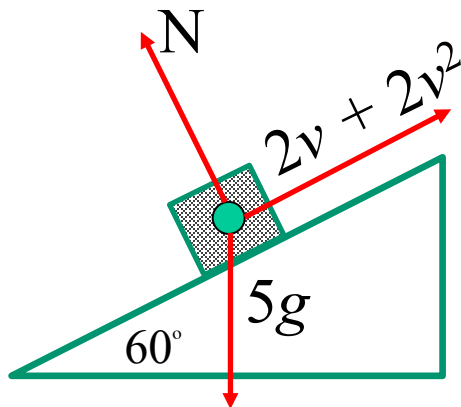

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(v) 2021 Extension 2 HSC Question 14 b)

An object of mass 5 kg is on a slope that is inclined at an angle of  $60^\circ$  to the horizontal. The acceleration due to gravity is  $g \text{ ms}^{-2}$  and the velocity of the object down the slope is  $v \text{ ms}^{-1}$ .

As well as the force due to gravity, the object is acted upon by two forces, one of magnitude  $2v$  newtons and one of magnitude  $2v^2$  newtons, both acting up the slope.

a) Show that the resultant force down the slope is



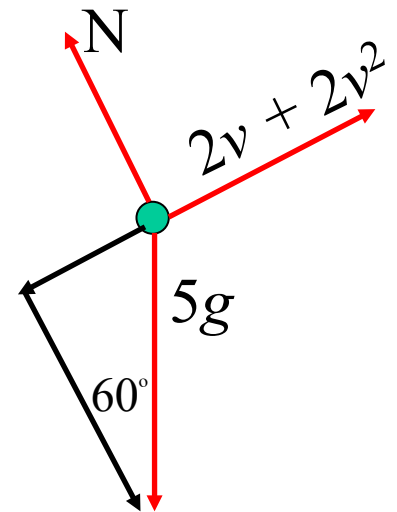
$$\frac{5\sqrt{3}}{2}g - 2v - 2v^2 \text{ newtons.}$$

resolving forces down the plane

$$m\ddot{x} = 5g \sin 60^\circ - 2v - 2v^2$$

$$= \frac{5\sqrt{3}}{2}g - 2v - 2v^2$$

---



b) There is one value of  $v$  such that the object will slide down the slope at a constant speed.

Find this value of  $v$  in  $\text{ms}^{-1}$ , correct to 1 decimal place, given that  $g = 10$ .

If the object slides down the slope at a constant speed,  $\ddot{x} = 0$ .

$$\text{i.e. } \frac{5\sqrt{3}}{2}g - 2v - 2v^2 = 0$$

$$25\sqrt{3} - 2v - 2v^2 = 0$$

$$v = \frac{2 \pm \sqrt{4 + 200\sqrt{3}}}{-4}$$

$$\begin{aligned} \text{but } v > 0 ; v &= \frac{-2 - \sqrt{4 + 200\sqrt{3}}}{-4} \\ &= \underline{4.2 \text{ ms}^{-1}} \quad (\text{to 1 dp}) \end{aligned}$$

**Exercise 6D;**  
**1, 2, 4, 5, 6, 8, 10, 12**

**Exercise 6E;**  
**1 to 8, 10, 13**