Binomial Expansions

A binomial expression is one which contains two terms.

$$(1+x)^0 = 1$$

$$(1+x)^1 = 1 + 1x$$

$$(1+x)^2 = 1 + 2x + 1x^2$$

$$\begin{aligned}(1+x)^3 &= (1+x)(1+2x+1x^2) \\ &= 1+2x+x^2+x+2x^2+x^3 \\ &= 1+3x+3x^2+x^3\end{aligned}$$

$$\begin{aligned}(1+x)^4 &= (1+x)(1+3x+3x^2+x^3) \\ &= 1+3x+3x^2+x^3+x+3x^2+3x^3+x^4 \\ &= 1+4x+6x^2+4x^3+x^4\end{aligned}$$

Blaise Pascal saw a pattern which we now know as **Pascal's Triangle**

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

1 11 55 165 330 462 462 330 165 55 11 1

1 12 66 220 495 792 924 792 495 220 66 12 1

$$e.g.(i)\left(1+\frac{2x}{3}\right)^7$$

$$=1^7+7(1)^6\left(\frac{2x}{3}\right)+21(1)^5\left(\frac{2x}{3}\right)^2+35(1)^4\left(\frac{2x}{3}\right)^3+35(1)^3\left(\frac{2x}{3}\right)^4+21(1)^2\left(\frac{2x}{3}\right)^5$$

$$+7(1)\left(\frac{2x}{3}\right)^6+\left(\frac{2x}{3}\right)^7$$

$$=1+\frac{14x}{3}+\frac{84x^2}{9}+\frac{280x^3}{27}+\frac{560x^4}{81}+\frac{672x^5}{243}+\frac{448x^6}{729}+\frac{128x^7}{2187}$$

(ii) Use the expansion of $(1-x)^{10}$ to find the value of $(0.998)^{10}$ to 8 dps

$$(1-x)^{10}=1-10x+45x^2-120x^3+210x^4-252x^5+210x^6-120x^7+45x^8$$

$$-10x^9+x^{10}$$

$$(0.998)^{10}=1-10(0.002)+45(0.002)^2-120(0.002)^3$$

$$=0.98017904$$

(iii) Find the coefficient of x^2 in $(2-3x)(4+5x)^4$

$$(2-3x)(4+5x)^4$$
$$=(2-3x)\left(4^4+4(4)^3(5x)+6(4)^2(5x)^2+4(4)(5x)^3+(5x)^4\right)$$

$$\therefore \text{coefficient of } x^2 = 2(6)(4)^2(5)^2-3(4)(4)^3(5)$$
$$=4800-3840$$
$$=960$$

Exercise 15A; 1*, 2ace etc, 4, 5, 6, 8a, 10b,
11ac, 12b, 13ac, 15a, 16a, 17, 23

**** question 1 we did in the theory***