

Binomial Expansions

A binomial expression is one which contains two terms.

$$(1+x)^0 = 1$$

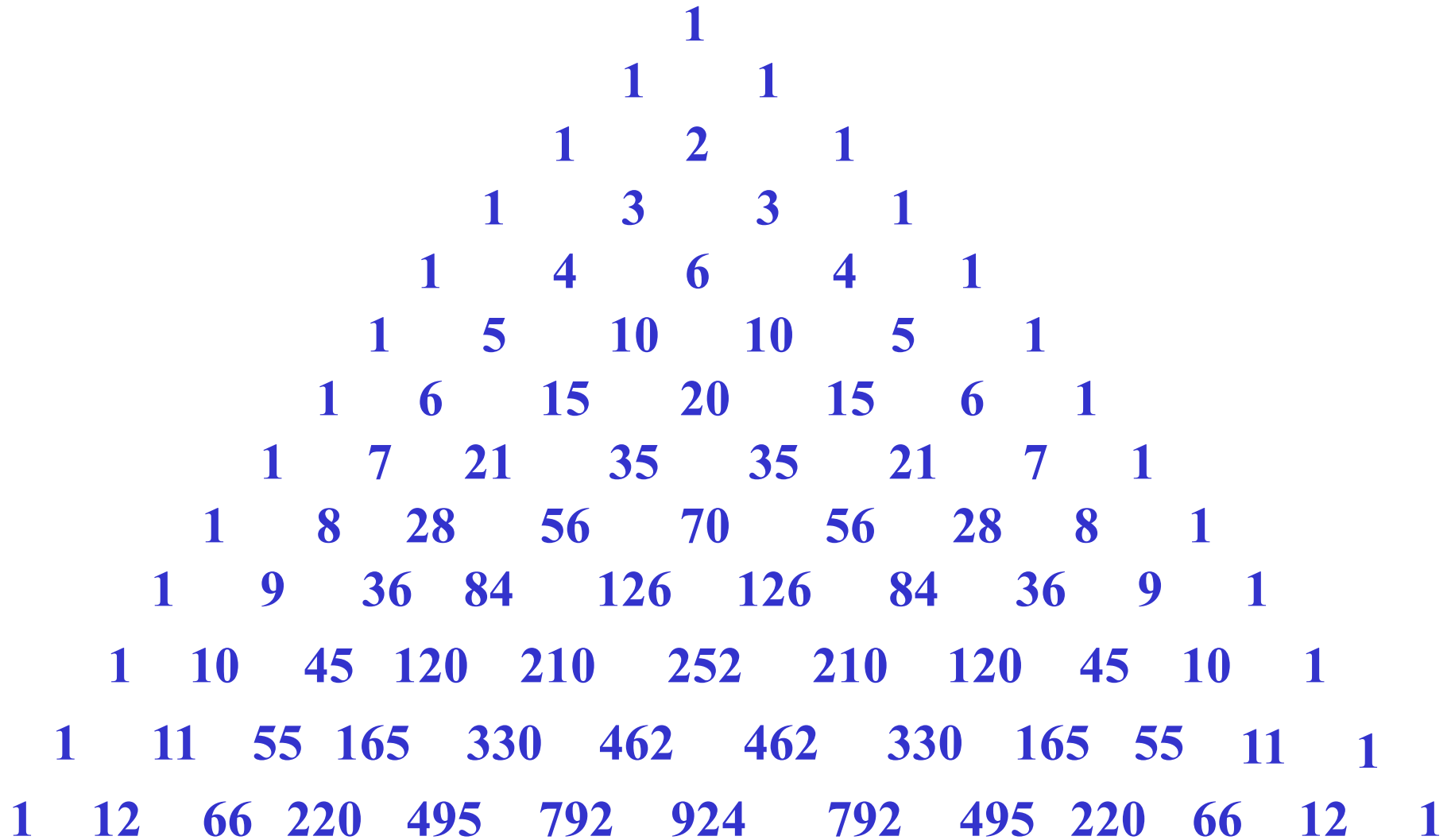
$$(1+x)^1 = 1+1x$$

$$(1+x)^2 = 1+2x+1x^2$$

$$\begin{aligned}(1+x)^3 &= (1+x)(1+2x+1x^2) \\ &= 1+2x+x^2+x+2x^2+x^3 \\ &= 1+3x+3x^2+x^3\end{aligned}$$

$$\begin{aligned}(1+x)^4 &= (1+x)(1+3x+3x^2+x^3) \\ &= 1+3x+3x^2+x^3+x+3x^2+3x^3+x^4 \\ &= 1+4x+6x^2+4x^3+x^4\end{aligned}$$

Blaise Pascal saw a pattern which we now know as **Pascal's Triangle**



Pascal's Triangle is a triangular array of binomial coefficients. Each number is the sum of the two numbers directly above it. The triangle is symmetric about its center. The numbers in each row correspond to the binomial coefficients for that row's index.

						1																								
						1		1																						
						1		2		1																				
						1		3		3		1																		
						1		4		6		4		1																
						1		5		10		10		5		1														
						1		6		15		20		15		6		1												
						1		7		21		35		35		21		7		1										
						1		8		28		56		70		56		28		8		1								
						1		9		36		84		126		126		84		36		9		1						
						1		10		45		120		210		252		210		120		45		10		1				
						1		11		55		165		330		462		462		330		165		55		11		1		
						1		12		66		220		495		792		924		792		495		220		66		12		1

$$\text{e.g. (i)} \left(1 + \frac{2x}{3}\right)^7$$

$$= 1^7 + 7(1)^6 \left(\frac{2x}{3}\right) + 21(1)^5 \left(\frac{2x}{3}\right)^2 + 35(1)^4 \left(\frac{2x}{3}\right)^3 + 35(1)^3 \left(\frac{2x}{3}\right)^4 + 21(1)^2 \left(\frac{2x}{3}\right)^5$$

$$+ 7(1) \left(\frac{2x}{3}\right)^6 + \left(\frac{2x}{3}\right)^7$$

$$= 1 + \frac{14x}{3} + \frac{84x^2}{9} + \frac{280x^3}{27} + \frac{560x^4}{81} + \frac{672x^5}{243} + \frac{448x^6}{729} + \frac{128x^7}{2187}$$

(ii) Use the expansion of $(1-x)^{10}$ to find the value of $(0.998)^{10}$ to 8 dps

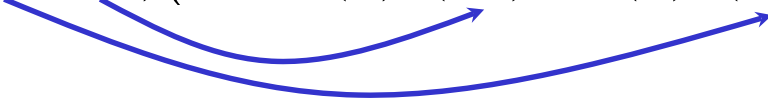
$$(1-x)^{10} = 1 - 10x + 45x^2 - 120x^3 + 210x^4 - 252x^5 + 210x^6 - 120x^7 + 45x^8 - 10x^9 + x^{10}$$

$$(0.998)^{10} = 1 - 10(0.002) + 45(0.002)^2 - 120(0.002)^3$$

$$= \underline{0.98017904}$$

(iii) Find the coefficient of x^2 in $(2 - 3x)(4 + 5x)^4$

$$(2 - 3x)(4 + 5x)^4$$

$$= (2 - 3x)(4^4 + 4(4)^3(5x) + 6(4)^2(5x)^2 + 4(4)(5x)^3 + (5x)^4)$$


$$\therefore \text{coefficient of } x^2 = 2(6)(4)^2(5)^2 - 3(4)(4)^3(5)$$

$$= 4800 - 3840$$

$$= \underline{960}$$

Exercise 15A; 1*, 2ace etc, 4, 5, 6, 8a, 10b,

11ac, 12b, 13ac, 15a, 16a, 17, 23

** question 1 we did in the theory*