The Binomial Theorem

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_k x^k + \ldots + {}^nC_n x^n$$

$$= \sum_{k=0}^{n} {}^nC_k x^k$$

where ${}^nC_k = \dfrac{n!}{k!(n-k)!}$ and n is a positive integer

NOTE: there are $(n + 1)$ terms

This extends to;

$$(a+b)^n = \sum_{k=0}^{n} {}^nC_k a^{n-k} b^k$$

e.g. Evaluate ${}^{11}C_4$

$$ {}^{11}C_4 = \frac{11!}{4!7!}$$

$$= \frac{11 \times 10 \times \cancel{9}^{3} \times \cancel{8}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 330$$

(ii) Find the value of n so that;

a) ${}^{n}C_5 = {}^{n}C_8$

$${}^{n}C_k = {}^{n}C_{n-k}$$

$$\therefore 8 = n - 5$$

$$\underline{n = 13}$$

b) ${}^{n}C_7 + {}^{n}C_8 = {}^{20}C_8$

$${}^{n-1}C_k + {}^{n-1}C_{k-1} = {}^{n}C_k$$

$$\therefore \underline{n = 19}$$

(iii) Find the 5th term in the expansion of $\left(5a - \frac{3}{b}\right)^{11}$

$$T_{k+1} = {}^{11}C_k (5a)^{11-k} \left(-\frac{3}{b}\right)^k$$

$$T_5 = {}^{11}C_4 (5a)^7 \left(-\frac{3}{b}\right)^4$$

$$= \underline{\frac{{}^{11}C_4 5^7 3^4 a^7}{b^4}} \quad \leftarrow \textit{unsimplified}$$

$$= \frac{330 \times 78125 \times 81 a^7}{b^4}$$

$$= \underline{\frac{2088281250 a^7}{b^4}}$$

(iv) Obtain the term independent of x in $\left(3x^2 - \frac{1}{2x}\right)^9$

$$T_{k+1} = {}^9C_k \left(3x^2\right)^{9-k} \left(-\frac{1}{2x}\right)^k$$

term independent of x means term with x^0

$$\left(x^2\right)^{9-k}\left(x^{-1}\right)^k = x^0$$

$$x^{18-2k} \times x^{-k} = x^0$$

$$18 - 3k = 0$$

$$k = 6$$

$$T_7 = {}^9C_6 \left(3x^2\right)^3 \left(-\frac{1}{2x}\right)^6$$

$$= \frac{{}^9C_6 3^3}{2^6}$$

(v) Find the ratio of the coefficients of x^7 and x^5 in the expansion of $(1 + 2x)^{15}$

$$T_{k+1} = \binom{15}{k}(2x)^k$$

$$\frac{\text{coefficient } x^7}{\text{coefficient } x^5} = \frac{\binom{15}{7}2^7}{\binom{15}{5}2^5}$$

$$= \frac{15!}{7!8!} \times \frac{5!10!}{15!} \times 4$$

$$= \frac{10 \times 9 \times 4}{7 \times 6}$$

$$= \frac{60}{7}$$

Exercise 15C;
5, 6, 7, 8a, 9b, 10a, 11b, 12, 14a, 15, 16, 18, 20

Exercise 15E;
1bdf, 2, 4, 5ac, 6ace, 7ad, 8ac, 9a, 10