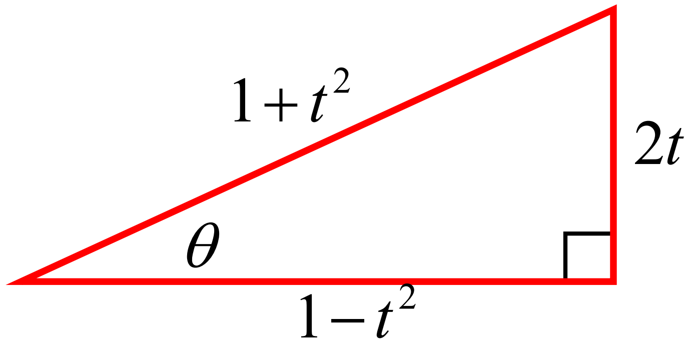
The t results

Let $t = \tan\frac{\theta}{2}$

$$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$\tan\theta = \frac{2t}{1-t^2}$$



$$h^2 = (2t)^2 + (1-t^2)^2$$
$$= 4t^2 + 1 - 2t^2 + t^4$$
$$= t^4 + 2t^2 + 1$$
$$= (t^2+1)^2$$

If $t = \tan\frac{\theta}{2}$;

$$\tan\theta = \frac{2t}{1-t^2} \qquad \sin\theta = \frac{2t}{1+t^2} \qquad \cos\theta = \frac{1-t^2}{1+t^2}$$

Note:

If $t = \tan\theta$; $\tan 2\theta = \frac{2t}{1-t^2}$

If $t = \tan 2\theta$; $\tan 4\theta = \frac{2t}{1-t^2}$

e.g. (i) Show that $\dfrac{1-\cos x}{\sin x} = t$, where $t = \tan\dfrac{x}{2}$

$$\frac{1-\cos x}{\sin x} = \frac{1-\dfrac{1-t^2}{1+t^2}}{\dfrac{2t}{1+t^2}}$$

$\left(\begin{array}{c} x \text{ is double } \dfrac{x}{2}, \\ \text{so } t \text{ results can be used for } \sin x, \cos x \end{array}\right)$

$$= \frac{1+t^2-\left(1-t^2\right)}{2t}$$

$$= \frac{2t^2}{2t}$$

$$= t$$

(ii) Use $t = \tan\dfrac{\theta}{2}$ to simplify $\dfrac{2\tan 75^\circ}{1+\tan^2 75^\circ}$

Let $t = \tan 75^\circ$;

i.e. $\dfrac{\theta}{2} = 75^\circ$

$\theta = 150^\circ$

$$\frac{2\tan 75^\circ}{1+\tan^2 75^\circ} = \frac{2t}{1+t^2}$$

$$= \sin\theta$$

$$= \sin 150^\circ$$

$$= \frac{1}{2}$$

(iii) Prove $\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \tan\dfrac{\theta}{2}$

Let $t = \tan\dfrac{\theta}{2}$;

$$\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \frac{1+\dfrac{2t}{1+t^2}-\dfrac{1-t^2}{1+t^2}}{1+\dfrac{2t}{1+t^2}+\dfrac{1-t^2}{1+t^2}}$$

$$= \frac{1+t^2+2t-\left(1-t^2\right)}{1+t^2+2t+1-t^2}$$

$$= \frac{2t^2+2t}{2+2t}$$

$$= \frac{2t(t+1)}{2(1+t)} = t = \tan\frac{\theta}{2}$$

(iv) By making the substitution $t = \tan\frac{\theta}{2}$ or otherwise,

show that $\text{cosec}\,\theta + \cot\theta = \cot\frac{\theta}{2}$ 2005 Extension 1 HSC Q4b)

$$\text{cosec}\,\theta + \cot\theta = \frac{1+t^2}{2t} + \frac{1-t^2}{2t}$$

$$= \frac{2}{2t}$$

$$= \frac{1}{t}$$

$$= \cot\frac{\theta}{2}$$

Exercise 17F; 1def, 3ace, 4bcf, 5aceg, 6, 8, 10bd, 12