Solving Inequations

All inequations can be solved using two simple steps;

1. Find the **critical points** (boundaries) of the solution by solving the corresponding equation
2. **Test the regions** between the critical points to see if whether or not they are included in the solution

1. Linear Inequations

Solve like a normal equation, remembering to change the sign if you multiply or divide by a negative number.

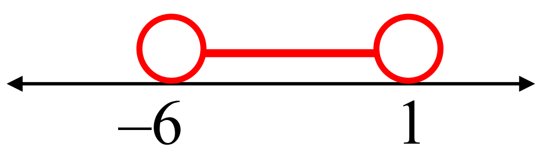
> *"if you change the sign, you change the sign"*

2. Quadratic (*and polynomials in general*) Inequations

e.g. $6 - 5x - x^2 > 0$

$x^2 + 5x - 6 < 0$

$(x + 6)(x - 1) < 0$



$-6 < x < 1$

3. Absolute Value Inequations

Solve using the definition of absolute value

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

e.g. $|3x + 2| \geq 1$

$$3x + 2 \geq 1 \quad \text{or} \quad -(3x+2) \geq 1$$

$$3x \geq -1 \qquad -3x - 2 \geq 1$$

$$x \geq -\frac{1}{3} \qquad -3x \geq 3$$

$$x \leq -1$$

$$\therefore x \leq -1 \quad \text{or} \quad x \geq -\frac{1}{3}$$

4. Inequations with Pronumerals in the Denominator

e.g. $\dfrac{2}{x+3} < 5$

$$x + 3 \neq 0$$

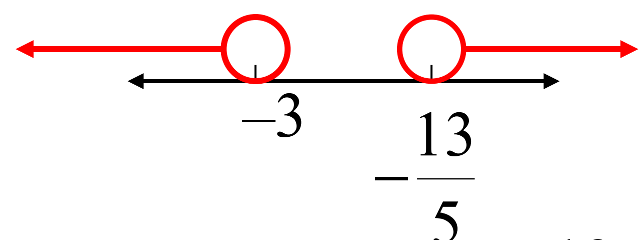
$$x \neq -3$$

$$\frac{2}{x+3} = 5$$

$$2 = 5x + 15$$

$$5x = -13$$

$$x = -\frac{13}{5}$$



$$\therefore x < -3 \quad \text{or} \quad x > -\frac{13}{5}$$

Note: 3 & 4 can be turned into turn it into a quadratic inequation

$$|3x + 2| \geq 1$$

square both sides

(squares are always positive, just like absolute value)

$$(3x+2)^2 \geq 1$$

$$9x^2 + 12x + 4 \geq 1$$

$$9x^2 + 12x + 3 \geq 0$$

$$3x^2 + 4x + 1 \geq 0$$

$$(3x+1)(x+1) \geq 0$$

$$\therefore x \leq -1 \quad \text{or} \quad x \geq -\frac{1}{3}$$

$$(ii)\ \frac{2}{x+3} < 5$$

$$2(x+3) < 5(x+3)^2$$

$$5(x+3)^2 - 2(x+3) > 0$$

$$(x+3)(5x+13) > 0$$

$$\therefore x < -3 \quad \text{or} \quad x > -\frac{13}{5}$$

multiply both sides by the denominator squared

(to ensure it is a positive number, so the sign stays the same)

(*iii)* Extension 1 HSC 2022 Q11 f)

$$\frac{x}{2-x} \geq 5$$

$$\frac{x}{2-x} - 5 \geq 0$$

$$\frac{x-(10-5x)}{2-x} \geq 0$$

$$\frac{6x-10}{2-x} \geq 0$$

Case 1: both numerator & denominator are positive

$$6x-10 \geq 0 \wedge 2-x > 0$$

$$x \geq \frac{5}{3} \wedge x < 2$$

$$\frac{5}{3} \leq x < 2$$

Case 2: both numerator & denominator are negative

$$6x-10 \leq 0 \wedge 2-x < 0$$

$$x \leq \frac{5}{3} \wedge x > 2$$

no solutions

$$\therefore \frac{5}{3} \leq x < 2$$

Using Graphs to Solve Inequations

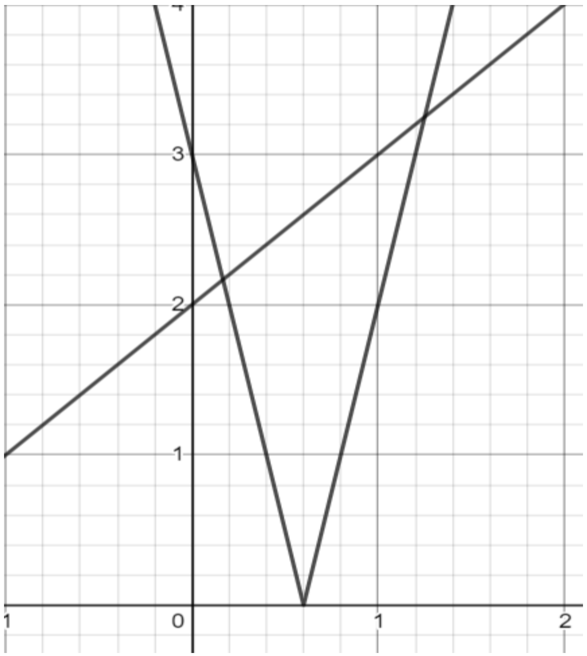
When we solved LHS – RHS > 0 graphically, we were essentially asking;

when is y = LHS – RHS above the line y=0 ?

$f(x) > g(x)$ can be solved graphically, using the same idea

when is y = f(x) above the line y = g(x) ?

computer graphing packages such as Desmos make this option more attractive



e.g. $|5x - 3| > x + 2$

$$|5x - 3| = x + 2$$

$$5x - 3 = x + 2 \qquad 3 - 5x = x + 2$$

$$4x = 5 \qquad 6x = 1$$

$$x = \frac{5}{4} \qquad x = \frac{1}{6}$$

$$x < \frac{1}{6} \quad \text{or} \quad x > \frac{5}{4}$$

Exercise 3D; 4c, 6e, 7d, 8c, 9b, 12b(i), 13c, 14a, 17, 18
Exercise 3E; 1bdf, 2bdf, 3c, 4c, 9d, 10d, 11b, 13bdf, 15, 16, 19bc, 20, 22