

Properties of Complex Conjugates

$$(1) |z| = |\bar{z}|$$

$$(2) \arg z = -\arg \bar{z}$$

$$(3) z\bar{z} = x^2 + y^2 \\ = |z|^2$$

$$(4) \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$$

$$(5) \overline{z_1 z_2} = \bar{z}_1 \times \bar{z}_2$$

$$(6) \overline{\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}} = \begin{pmatrix} \bar{z}_1 \\ \bar{z}_2 \end{pmatrix}$$

$$(7) \frac{1}{z} = \frac{\bar{z}}{|z|^2}$$

e.g. If $x + iy = \sqrt{\frac{6+2i}{3-i}}$, show that $x^2 + y^2 = 2$

$$x + iy = \sqrt{\frac{6+2i}{3-i}}$$

$$(x + iy)^2 = \frac{6+2i}{3-i} \dots (1)$$

$$\overline{(x + iy)^2} = \overline{\left(\frac{6+2i}{3-i}\right)}$$

$$= \frac{\overline{6+2i}}{\overline{3-i}}$$

$$(x - iy)^2 = \frac{6-2i}{3+i} \dots (2)$$

Multiply (1) \times (2)

$$(x + iy)^2 (x - iy)^2 = \frac{6+2i}{3-i} \times \frac{6-2i}{3+i}$$

$$(x^2 + y^2)^2 = \frac{36+4}{9+1}$$

$$= 4$$

$$\underline{x^2 + y^2 = 2}$$

Patel;
Exercise 4H; 1 to 6