Integrating Trig

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b)+c$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b)+c$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b)+c$$

e.g. $(i)\ \int \sin 3x\,dx = -\frac{1}{3}\cos 3x + c$

$(ii)\ \int \cos(1-5x)\,dx = -\frac{1}{5}\sin(1-5x)+c$

$(iii)\ \int \sec^2\left(\frac{x}{2}\right)dx = 2\tan\left(\frac{x}{2}\right)+c$

$$(iv)\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= -\frac{1}{2}\left(\cos\pi - \cos\frac{\pi}{3}\right)$$

$$= -\frac{1}{2}\left(-1-\frac{1}{2}\right)$$

$$= \frac{3}{4}$$

$$(v)\int x\sec^2 x^2\,dx = \frac{1}{2}\int 2x\sec^2 x^2\,dx$$

$$= \frac{1}{2}\tan x^2 + c$$

$$(vi)\int \sin^2 x\,dx$$

$$= \frac{1}{2}\int(1-\cos 2x)\,dx$$

$$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + c$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + c$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 1 - 2\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$= 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$(vii)\int \tan x\,dx$$

$$= \int \frac{\sin x}{\cos x}\,dx$$

$$= -\int \frac{-\sin x}{\cos x}\,dx$$

$$= -\log|\cos x| + c$$

$$= \log|\cos x|^{-1} + c$$

$$= \log|\sec x| + c$$

$$(viii)\int_0^{\frac{\pi}{2}} \cos x \sin^7 x\,dx$$

$$u = \sin x$$

$$du = \cos x\,dx$$

$$x = 0, u = 0$$

$$x = \frac{\pi}{2}, u = 1$$

$$= \int_0^1 u^7\,du$$

$$= \left[\frac{1}{8}u^8\right]_0^1$$

$$= \frac{1}{8}(1^8 - 0)$$

$$= \frac{1}{8}$$

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$$\int_0^{\frac{\pi}{2}} \cos 5x \sin 3x \, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \{\sin(-2x) + \sin 8x\} dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 8x - \sin 2x) dx$$

$$= \frac{1}{2}\left[-\frac{1}{8}\cos 8x + \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left(-\frac{1}{8} - \frac{1}{2} + \frac{1}{8} - \frac{1}{2}\right)$$

$$= -\frac{1}{2}$$

Exercise 7D;
1ain, 2bdfh, 6eg, 7, 8c, 9d, 10, 11, 12b, 13a, 14b, 15a, 16ace, 17, 18, 20b i

Exercise 7E;
5bdf, 6b, 11, 12, 13, 18, 20, 21, 22