

Integration Involving Inverse Trig

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c \quad \text{OR} \quad -\cos^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1}\frac{f(x)}{a} + c \quad \text{OR} \quad -\cos^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1}\frac{f(x)}{a} + c$$

e.g.

$$(i) \int \frac{dx}{\sqrt{4-x^2}}$$
$$= \sin^{-1}\left(\frac{x}{2}\right) + c$$

$$(ii) \int \frac{dx}{9+x^2}$$
$$= \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + c$$

$$(iii) \int_0^1 \frac{dx}{\sqrt{2-x^2}} = \left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) \right]_0^1$$
$$= \sin^{-1} \frac{1}{\sqrt{2}} - \sin^{-1} 0$$
$$= \frac{\pi}{4} - 0$$
$$= \frac{\pi}{4}$$

(iv) Find $\frac{d}{dx} \{\sin^{-1} e^x\}$ and hence evaluate $\int_{-\log 2}^0 \frac{e^x}{\sqrt{1-e^{2x}}} dx$

$$\frac{d}{dx} \{\sin^{-1} e^x\} = \frac{e^x}{\sqrt{1-e^{2x}}}$$

$$\begin{aligned} \therefore \int_{-\log 2}^0 \frac{e^x}{\sqrt{1-e^{2x}}} dx &= [\sin^{-1} e^x]_{-\log 2}^0 \\ &= \sin^{-1} e^0 - \sin^{-1} e^{-\log 2} \\ &= \sin^{-1} 1 - \sin^{-1} \frac{1}{2} \end{aligned}$$

$$= \frac{\pi}{2} - \frac{\pi}{6}$$

$$= \frac{\pi}{3}$$

$$\begin{aligned} (v) \int \frac{dx}{\sqrt{4-9x^2}} &= \frac{1}{3} \int \frac{3 dx}{\sqrt{4-9x^2}} \\ &= \frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right) + c \end{aligned}$$

Exercise 12B; 2, 3, 4a, 5b, 6bdf, 7bd, 8, 11, 13, 14, 15, 18, 19, 20, 22, 23