Integrating Derivative on Function

$$\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$$

e.g. (*i*) $\int \frac{1}{7-3x}\,dx$

$= -\frac{1}{3}\int \frac{-3}{7-3x}\,dx$

$= -\frac{1}{3}\log|7-3x| + c$

(*ii*) $\int \frac{dx}{8x+5}$

$= \frac{1}{8}\int \frac{8dx}{8x+5}$

$= \frac{1}{8}\log|8x+5| + c$

(*iii*) $\int \frac{x^5\,dx}{x^6+2}$

$= \frac{1}{6}\int \frac{6x^5\,dx}{x^6+2}$

$= \frac{1}{6}\log(x^6+2) + c$

$$(iv)\ \int \frac{1}{5x}\,dx$$

$$= \frac{1}{5}\int \frac{5}{5x}\,dx$$

$$= \frac{1}{5}\log|5x| + c$$

OR

$$\frac{1}{5}\int \frac{1}{x}\,dx$$

$$= \frac{1}{5}\log|x| + c$$

$$(v)\ \int \frac{4x+1}{2x+1}\,dx$$

order numerator $\geq$ order denominator

$\Rightarrow$ polynomial division

$$= \int \left[2 - \frac{1}{2x+1}\right]dx$$

$$= 2x - \frac{1}{2}\log|2x+1| + c$$

$$\begin{array}{r} 2 \\ 2x+1\,\overline{)\,4x+1} \\ 4x+2 \\ \hline -1 \end{array}$$

$(vi)\ \int_1^2 \frac{2x}{x^2+1}dx$

$= \left[\log\left(x^2+1\right)\right]_1^2$

$= \log 5 - \log 2$

$= \log\left(\frac{5}{2}\right)$

Take care
with definite integrals
You CANNOT integrate across an asymptote

(vii) Differentiate $x^3 \log x$ and hence integrate $x^2 \log x$

$\frac{d}{dx}\left\{x^3 \log x\right\} = \left(x^3\right)\left(\frac{1}{x}\right) + (\log x)\left(3x^2\right)$

$= x^2 + 3x^2 \log x$

$\int (x^2 + 3x^2 \log x)dx = x^3 \log|x| + c$

$3\int x^2 \log x dx = x^3 \log|x| - \int x^2 dx + c$

$\int x^2 \log x dx = \frac{1}{3} x^3 \log|x| - \frac{1}{9} x^3 + c$

Exercise 6I; 1c, 2b I, 3acd, 4c, 5c, 6adf, 7, 8beh, 9c, 10a, 11bd, 12bf, 13bd, 14c, 15c

Exercise 6J; 3a, 4b, 5a, 6b, 7a, 8b, 9b, 1, 15 to 18, 20, 22abc