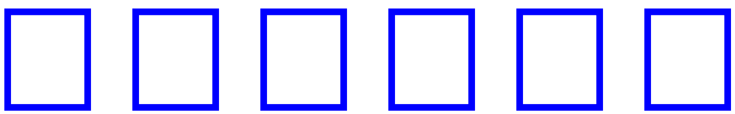
Permutations

Case 4: Ordered Sets of *n* Objects, Arranged in a Circle

What is the difference between placing objects in a line and placing objects in a circle?

The difference is the number of ways the first object can be placed.

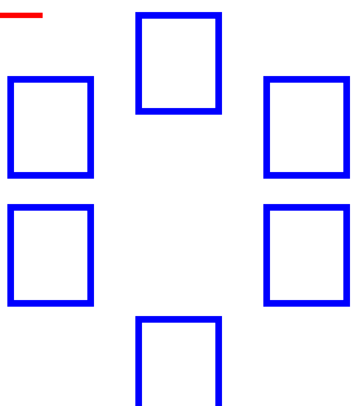
Line



In a line there is a definite start and finish of the line.

The first object has a choice of 6 positions

Circle



In a circle there is no definite start or finish of the circle.

It is not until the first object chooses its position that positions are defined.

Line

possibilities for object 1

possibilities for object 2

possibilities for object 3

possibilities for last object

$$\text{Number of Arrangements} = n \times (n-1) \times (n-2) \times \cdots \times 1$$

Circle

possibilities for object 1

possibilities for object 2

possibilities for object 3

possibilities for last object

$$\text{Number of Arrangements} = 1 \times (n-1) \times (n-2) \times \cdots \times 1$$

$$\text{Number of Arrangements in a circle} = \frac{n!}{n} = (n-1)!$$

e.g. A meeting room contains a round table surrounded by ten chairs.

(i) A committee of ten people includes three teenagers. How many arrangements are there in which all three sit together?

the number of ways the three teenagers can be arranged

number of ways of arranging 8 objects in a circle (3 teenagers) + 7 others

$$\text{Arrangements} = 3! \times 7!$$
$$= \underline{30240}$$

(ii) Elections are held for Chairperson and Secretary.
What is the probability that they are seated directly opposite each other?

$$\text{Ways (no restrictions)} = 9!$$

President can sit anywhere as they are 1st in the circle

Secretary must sit opposite President

Ways remaining people can go

$$\text{Ways (restrictions)} = 1 \times 1 \times 8!$$

$$P(\text{P \& S opposite}) = \frac{1 \times 1 \times 8!}{9!}$$

$$= \frac{1}{9}$$

Note: of 9 seats only 1 is opposite the President

$$\therefore P(opposite) = \frac{1}{9}$$

Sometimes simple logic is quicker!!!!

2002 Extension 1 HSC Q3a)

Seven people are to be seated at a round table

(i) How many seating arrangements are possible?

$$\text{Arrangements} = 6!$$
$$= \underline{720}$$

(ii) Two people, Kevin and Jill, refuse to sit next to each other. How many seating arrangements are then possible?

Note: it is easier to work out the number of ways Kevin and Jill are together and subtract from total number of arrangements.

the number of ways Kevin & Jill are together

number of ways of arranging 6 objects in a circle (Kevin & Jill) + 5 others

$$\text{Arrangements} = 2! \times 5!$$
$$= 240$$

$$\text{Arrangements} = 720 - 240$$
$$= \underline{480}$$

2023 Extension 1 HSC Q10

A group with 5 students and 3 teachers is to be arranged in a circle.

In how many ways can this be done if no more than 2 students can sit together?

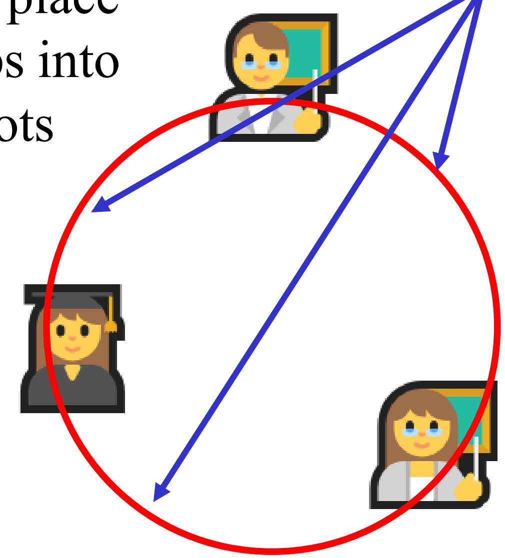
1. place the teachers in the circle

Divide by 2! as two groups are of the same size

2. this creates three spots for the students to be inserted into

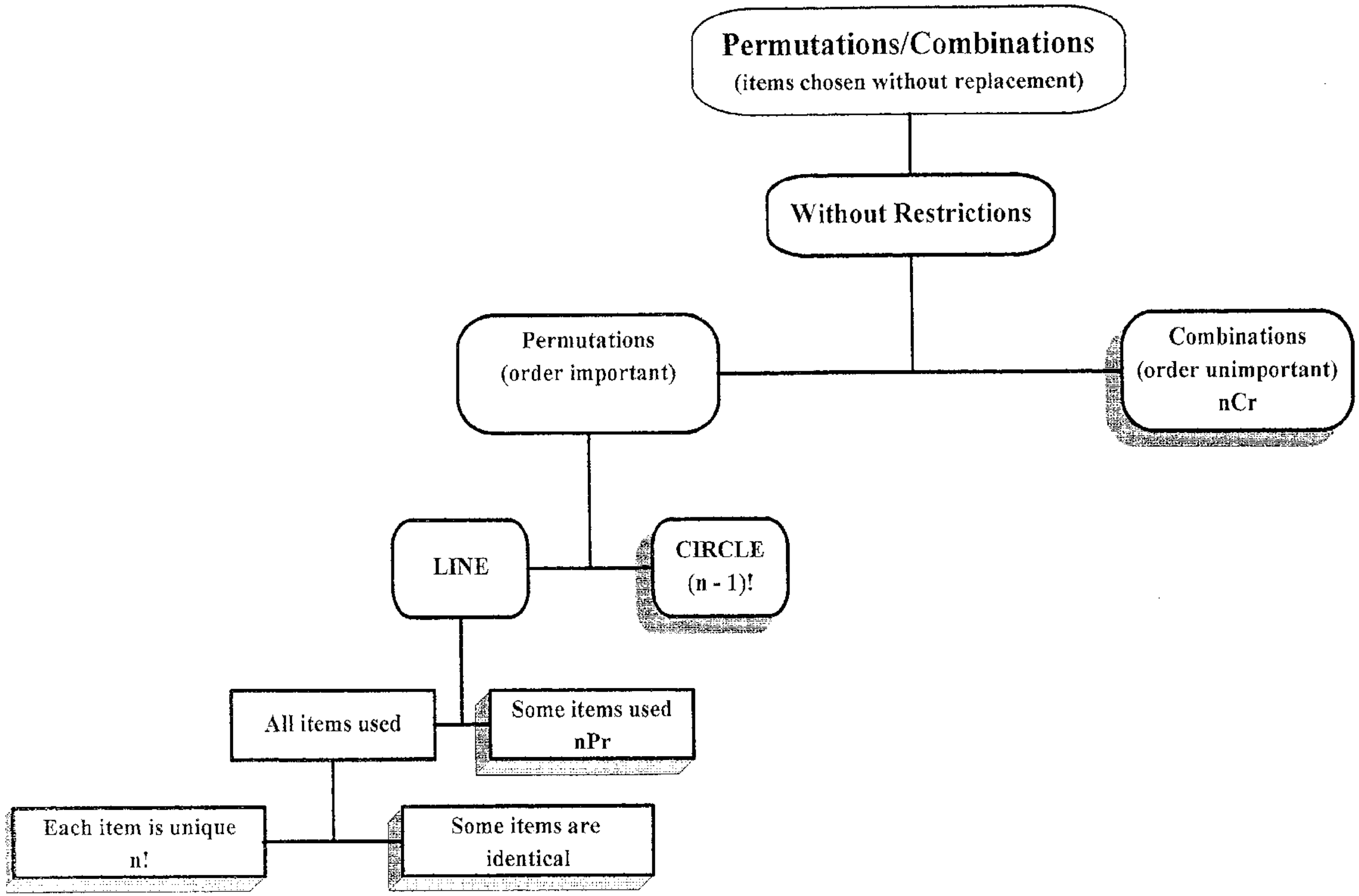
5. finally place the groups into the spots

$$\text{Ways} = 2! \times \frac{5!}{2!2!2!} \times 2!2! \times 3!$$
$$= 5! \times 3!$$
$$= 720$$



3. as no more than two can be together, the students must be organised into two groups of two and one solo student

4. now arrange the students in the two groups of two (as their order is important)


Permutations/Combinations
(items chosen without replacement)
Without Restrictions
Permutations
(order important)
Combinations
(order unimportant)
nCr
LINE
CIRCLE
(n - 1)!
All items used
Some items used
nPr
Each item is unique
n!
Some items are
identical

Exercise 14G; 1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 14