# *Investing Money by Regular Instalments*

## Annuities

An annuity is a compound interest investment from which payments are made or received on a regular basis for a fixed period of time.

**ordinary annuity:** deposit the same amount at the end of each compounding period e.g. loans

**annuity due:** deposit the same amount at the beginning of each compounding period e.g. superannuation fund

**future value (FV):** the total value of the investment at the end of the term of the investment.

**present value (PV):** the single amount that could be initially invested to produce the same FV over a given time period

*Note: annuities are not to be confused with the financial product known as an "annuity", whilst this product is a type of annuity, it is NOT the only type.*

# *Annuity Due*

2002 HSC Question 9b)

A superannuation fund pays an interest rate of 8.75% p.a. which compounds annually. Stephanie decides to invest $5000 in the fund at the beginning of each year, commencing on 1 January 2003.

a) What will be the value of Stephanie's superannuation when she retires on 31 December 2023?

$A_{21} = 5000(1.0875)^{21}$ amount invested for 21 years

$A_{20} = 5000(1.0875)^{20}$ amount invested for 20 years

$A_{19} = 5000(1.0875)^{19}$ amount invested for 19 years

$\vdots$

$\vdots$

$A_{1} = 5000(1.0875)^{1}$ amount invested for 1 year

$$\text{Amount} = 5000(1.0875)^{21} + 5000(1.0875)^{20} \ldots + 5000(1.0875)$$

$$a = 5000(1.0875), r = 1.0875, n = 21$$

$$= S_{21}$$

$$= \frac{5000(1.0875)(1.0875^{21} - 1)}{0.0875}$$

$$= \$299604.86$$

b*) Find the year when the fund first exceeds \$200000.

$$\text{Amount} = 5000(1.0875) + 5000(1.0875)^2 + \ldots + 5000(1.0875)^n$$

$$= S_n$$

i.e $S_n > 200000$

$$\frac{5000(1.0875)\left(1.0875^n - 1\right)}{0.0875} > 200000$$

$$\left(1.0875^n - 1\right) > \frac{280}{87}$$

$$1.0875^n > \frac{367}{87}$$

$$\log\left(1.0875^n\right) > \log\left(\frac{367}{87}\right)$$

$$n\log(1.0875) > \log\left(\frac{367}{87}\right)$$

$$n > \frac{\log\left(\frac{367}{87}\right)}{\log(1.0875)}$$

$$n > 17.16056585$$

$$\therefore n = 18$$

Thus 2021 is the first year when the fund exceeds $200000

c*) What annual instalment would have produced \$1000000 by 31st December 2020?

$$\text{Amount} = P(1.0875)^{18} + P(1.0875)^{17} + \ldots + P(1.0875)$$

$$a = P(1.0875), r = 1.0875, n = 18$$

i.e. $S_{18} = 1000000$

$$\frac{P(1.0875)\left(1.0875^{18} - 1\right)}{0.0875} = 1000000$$

$$P = \frac{(1000000)(0.0875)}{1.0875\left(1.0875^{18} - 1\right)}$$

$$= 22818.16829$$

An annual instalment of \$22818.17 will produce \$1000000

# *Future Value of Annuity Due*

$$FV = PR + PR^2 + PR^3 + \ldots PR^n$$

$$= \frac{PR(R^n - 1)}{R - 1}$$

$P =$ principal

$R = 1 +$ interest rate as a decimal(or fraction)

$n =$ time periods

d*) If Stephanie could afford it, how much money could she save by making one investment and still have the same future value in 21 years time?

$$PV(1.0875)^{21} = 299604.86$$

$$PV = \frac{299604.86}{(1.0875)^{21}}$$

$$= 54467.64$$

A present value of $54 467.64 would need to be invested

$$\text{actual investment} = \$5000 \times 21$$
$$= \$105\ 000$$

$$\text{saving} = 105000 - 54467.64$$
$$= 50532.36$$

Stephanie would save \$50 532.36

# *Present Value of Annuity Due*

$$(PV)R^n = FV$$

$$= \frac{PR(R^n - 1)}{R - 1}$$

$$PV = \frac{PR\left(1 - R^{-n}\right)}{R - 1}$$

$P =$ principal

$R = 1 +$ interest rate as a decimal(or fraction)

$n =$ time periods

(ii) A table of future value interest factors is used to compare annuities of $1

| Table of future value interest factors | | | | | | |
|---|---|---|---|---|---|---|
| Period | Interest rate per period | | | | | |
| | 0.50% | 1.50% | 2.00% | 3.00% | 4.00% | 6.00% |
| 4 | 4.0301 | 4.0909 | 4.1216 | 4.1836 | 4.2465 | 4.3746 |
| 8 | 8.1414 | 8.4328 | 8.5830 | 8.8923 | 9.2142 | 9.8975 |
| 16 | 16.6142 | 17.9324 | 18.6393 | 20.1569 | 21.8245 | 25.6725 |
| 32 | 34.6086 | 40.6883 | 44.2270 | 52.5028 | 62.7015 | 90.8898 |
| 48 | 54.0978 | 69.5652 | 79.3535 | 104.4084 | 139.2632 | 256.5645 |
| 96 | 122.8285 | 211.7202 | 284.6467 | 535.8502 | 1054.2960 | 4462.6505 |

Tim and Janene were offered interest of 6% p.a. for 4 years.

Tim invested $1500 each quarter compounded quarterly and Janene invested $500 every month compounded monthly.

Which person earned the greater amount of interest?

Tim: 6% p.a = 1.5% per quarter

$FV = 17.9324 \times 1500$

$= 26898.6$

Janene: 6% p.a = 0.5% per month

$FV = 54.0978 \times 500$

$= 27048.9$

$$\text{Tim's interest} = 26\,898.60 - 1500 \times 16$$
$$= 2\,898.60$$

$$\text{Janene's interest} = 27\,048.90 - 500 \times 48$$
$$= 3\,048.90$$

Janene earned \$150.30 more interest than Tim

**Exercise 14D; 5, 9, 12, 13, 17**