Polynomial Division

$$P(x) = A(x) \times Q(x) + R(x)$$

where;

$A(x)$ is the divisor

$Q(x)$ is the quotient

$R(x)$ is the remainder

Note:

degree $R(x)$ < degree $A(x)$

Q(x) and $R(x)$ are unique

1998 Extension 1 HSC Q2a)

Find the quotient, $Q(x)$, and the remainder, $R(x)$, when the polynomial $P(x) = x^4 - x^2 + 1$ is divided by $x^2 + 1$

$$
\begin{array}{r l}
 & x^2 \qquad\quad - 2 \\
x^2 + 1 \big) & \overline{x^4 + 0x^3 - x^2 + 0x + 1} \\
 & \underline{x^4 + 0x^3 + x^2} \\
 & \qquad\quad -2x^2 + 0x + 1 \\
 & \qquad\quad \underline{-2x^2 + 0x - 2} \\
 & \qquad\qquad\qquad\qquad 3
\end{array}
$$

$$\therefore Q(x) = x^2 - 2 \quad \text{and} \quad R(x) = 3$$

(i) Divide the polynomial 1988 Extension 1 HSC Q4b)

$$f(x) = 2x^4 - 10x^3 + 12x^2 + 2x - 3 \quad \text{by} \quad g(x) = x^2 - 3x + 1$$

$$\begin{array}{r l}
 & 2x^2 - 4x \quad - 2 \\
x^2 - 3x + 1 \big) & 2x^4 - 10x^3 + 12x^2 + 2x - 3 \\
 & \underline{2x^4 - 6x^3 + 2x^2} \\
 & \qquad -4x^3 + 10x^2 \quad + 2x - 3 \\
 & \qquad \underline{-4x^3 + 12x^2 - 4x} \\
 & \qquad\qquad\quad -2x^2 + 6x - 3 \\
 & \qquad\qquad\quad \underline{-2x^2 + 6x - 2} \\
 & \qquad\qquad\qquad\qquad -1
\end{array}$$

(ii) Hence write $f(x) = g(x)q(x) + r(x)$ where $q(x)$ and $r(x)$ are polynomials and $r(x)$ has degree less than 2.

$$\underline{2x^4 - 10x^3 + 12x^2 + 2x - 3 = \left(x^2 - 3x + 1\right)\left(2x^2 - 4x - 2\right) - 1}$$

Further graphing of polynomials

solve $R(x) = 0$ to find where (if anywhere) the curve cuts the horizontal/oblique asymptote

$$\boxed{y = \frac{P(x)}{A(x)} = Q(x) + \frac{R(x)}{A(x)}}$$

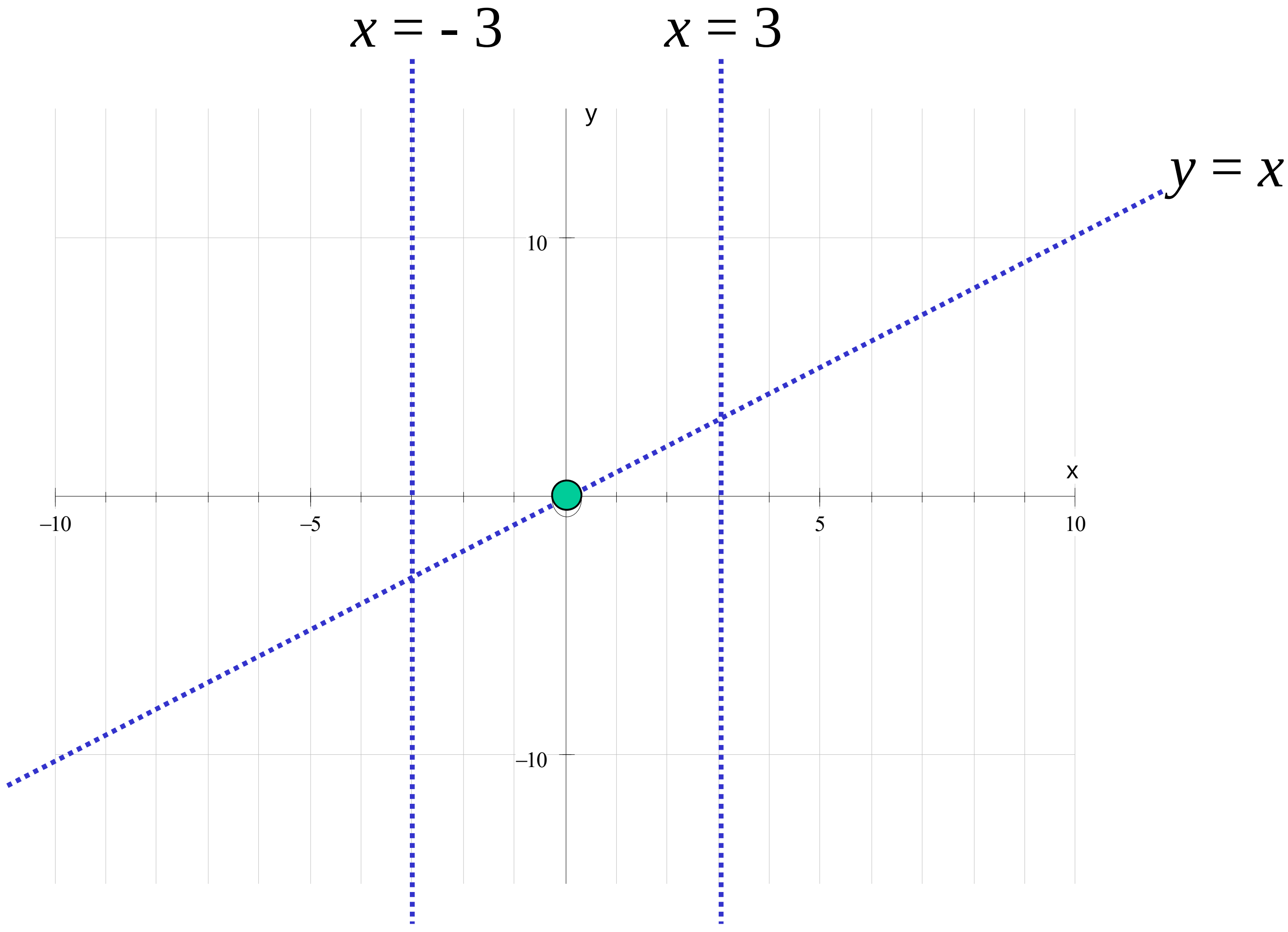
$y = Q(x)$ is the horizontal/oblique asymptote

solve $A(x) = 0$ to find vertical asymptotes

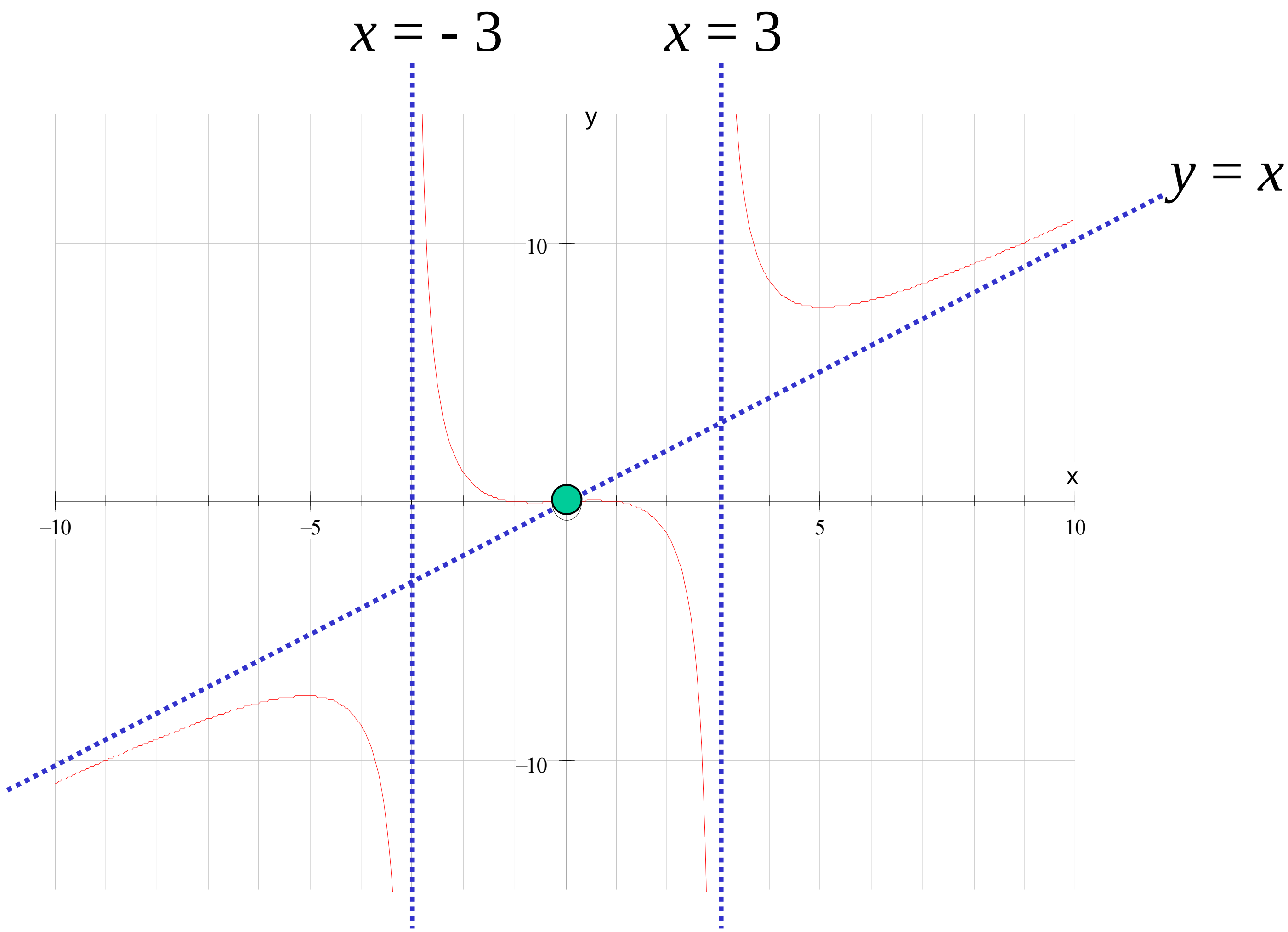
Example: Sketch the graph of $y = x + \dfrac{8x}{x^2 - 9}$, clearly indicating any asymptotes and any points where the graph meets the axes.

- vertical asymptotes at $x = \pm 3$
- oblique asymptote at $y = x$
- curve meets oblique asymptote at $8x = 0$

$$x = 0$$



x = - 3
x = 3
y
y = x
10
x
-10
-5
5
10
-10



x = - 3
x = 3
y = x
y
x
10
–10
–10
–5
5
10

Exercise 10C; 1c, 2bdfh, 3bd, 4ac, 6ace, 7, 9, 13