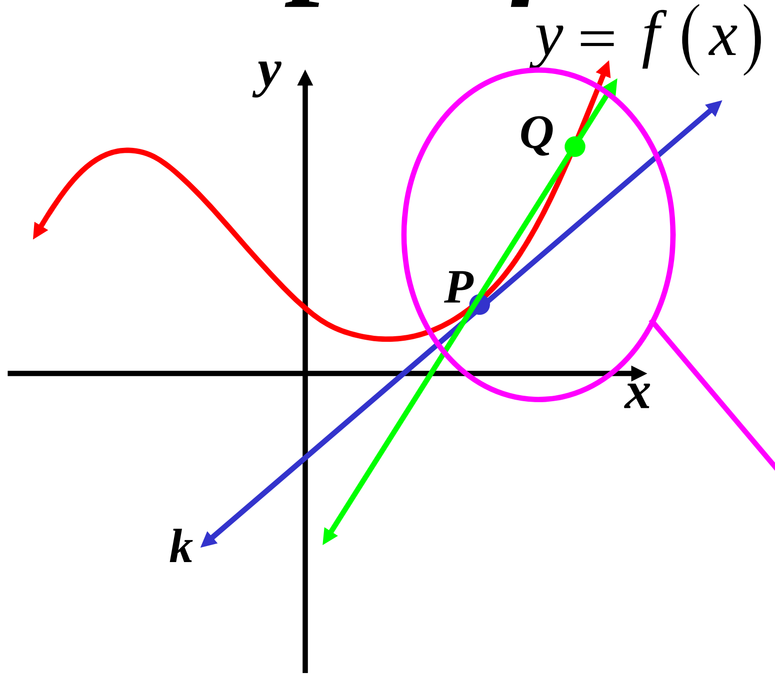
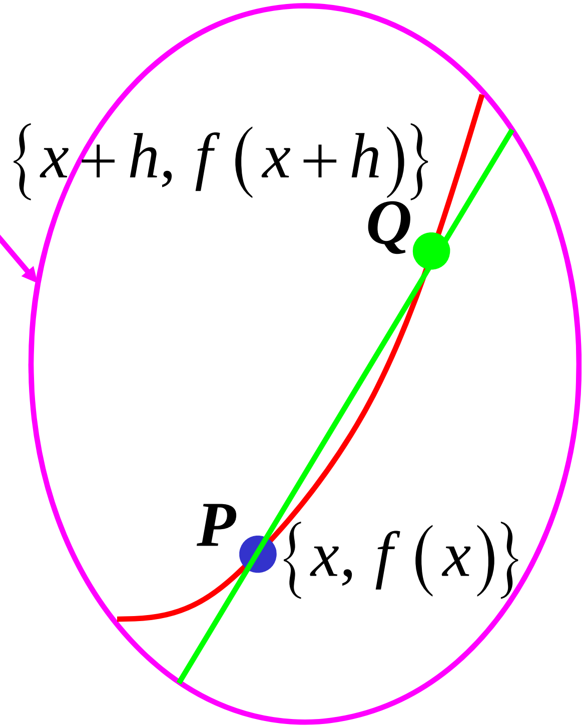
The Slope of a Tangent to a Curve



Slope ***PQ*** is an estimate
for the slope of line ***k***.

Q: Where do we position ***Q***
to get the best estimate?

A: As close to ***P*** as possible.



$$m_{PQ} = \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

To find the exact value of the slope of ***k***, we calculate the limit of the slope ***PQ*** as ***h*** gets closer to 0.

$$\boxed{\text{slope of tangent} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

This is known as the **"derivative of *y* with respect to *x*"** and is symbolised; $\frac{dy}{dx}$, y', $f'(x)$, $\frac{d}{dx}\{f(x)\}$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

the derivative measures the rate of something changing

The process is called **"differentiating from first principles"**

e.g. (i) Differentiate $y = 6x + 1$ by using first principles.

$$f(x) = 6x + 1$$

$$f(x+h) = 6(x+h) + 1$$
$$= 6x + 6h + 1$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{6x + 6h + 1 - (6x+1)}{h}$$
$$= \lim_{h \to 0} \frac{6h}{h}$$
$$= \lim_{h \to 0} 6$$
$$\underline{= 6}$$

(ii) Find the equation of the tangent to $y = x^2 - 5x + 2$ at the point $(1, -2)$.

$$f(x) = x^2 - 5x + 2$$
$$f(x+h) = (x+h)^2 - 5(x+h) + 2$$
$$= x^2 + 2xh + h^2 - 5x - 5h + 2$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h + 2 - x^2 + 5x - 2}{h}$$
$$= \lim_{h \to 0} \frac{2xh + h^2 - 5h}{h}$$
$$= \lim_{h \to 0} 2x + h - 5$$
$$= 2x - 5$$

when $x = 1$, $\dfrac{dy}{dx} = 2(1) - 5$

$$= -3$$

$\therefore$ the slope of the tangent at $(1, -2)$ is -3

$$y+2=-3(x-1)$$

$$y+2=-3x+3$$

$$y=-3x+1$$

Exercise 9B; 2, 3, 5a, 7ab (i, ii, v), 10, 11, 12, 13