Related Rates of Change

In some cases two, or more, rates must be found to get the equation in terms of the given variable.

$$\boxed{\frac{dy}{dt}=\frac{dy}{dx}\cdot\frac{dx}{dt}}$$

e.g. (i) 2017 Extension 1 HSC Q8

A stone drops into a pond, creating a circular ripple. The radius of the ripple increases from 0 cm, at a constant rate of 5 cms^{-1}. At what rate is the area enclosed within the ripple increasing when the radius is 15 cm?

$$\frac{dA}{dt}=? \qquad A=\pi r^2 \qquad \frac{dA}{dt}=\frac{dr}{dt}\times\frac{dA}{dr} \qquad \text{when } r=15,\ \frac{dA}{dt}=10\pi(15)$$

$$\frac{dr}{dt}=5 \qquad \frac{dA}{dr}=2\pi r \qquad =5\times 2\pi r \qquad =150\pi$$

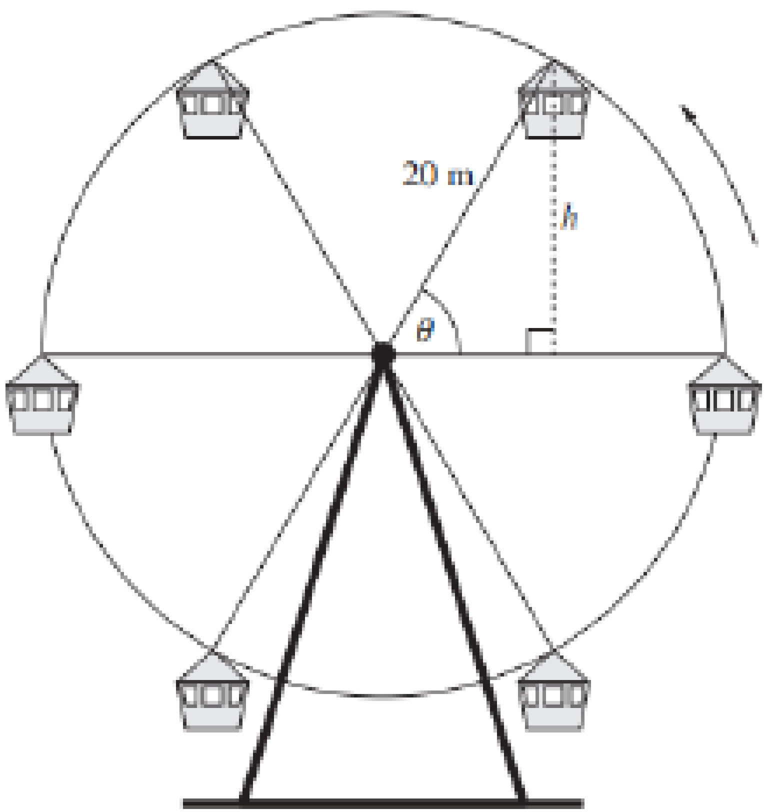
$$=10\pi r$$

$\therefore$ the ripple's area is increasing at a rate of 150π cm^{2}s^{-1}

(ii) 2018 Extension 1 HSC Q12 b)

A ferris wheel has a radius of 20 metres and is rotating at a rate of 1.5 radians per minute. The top of the carriage is h metres above the horizontal diameter of the ferris wheel.

The angle of elevation of the top of the carriage from the centre of the ferris wheel is θ



(i) Show that $\dfrac{dh}{d\theta} = 20\cos\theta$

$$\frac{h}{20} = \sin\theta$$

$$h = 20\sin\theta$$

$$\frac{dh}{d\theta} = 20\cos\theta$$

(ii) At what speed is the top of the carriage rising when it is 15 metres high than the horizontal diameter of the ferris wheel? Give your answer correct to one decimal place.

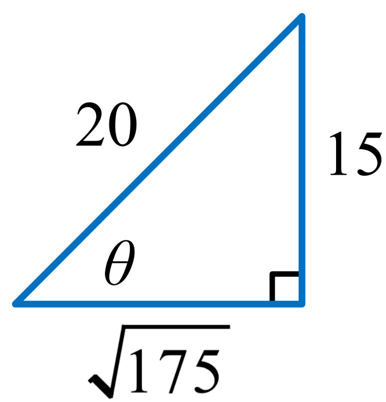
$$\frac{dh}{dt} = ? \qquad \frac{dh}{d\theta} = 20\cos\theta \qquad \frac{d\theta}{dt} = \frac{3}{2}$$

$$\frac{dh}{dt} = \frac{dh}{d\theta} \times \frac{d\theta}{dt}$$

$$= 20\cos\theta \times \frac{3}{2}$$

$$= 30\cos\theta$$

When $h = 15$; $\dfrac{dh}{dt} = 30\cos\theta$



$$= 30 \times \frac{\sqrt{175}}{20}$$

$$= 19.843\ldots$$

$\therefore$ the carriage is rising at a rate of 19.8 m/s

(iii) 2019 Extension 1 HSC Q12 a)

Distance A is inversely proportional to distance B, such that $A = \dfrac{9}{B}$, where A and B are measured in metres. The two distances vary with respect to time.

Distance B is increasing at a rate of 0.2 ms^{-1}.

What is the value of $\dfrac{dA}{dt}$ when $A = 12$?

$$\frac{dA}{dt} = ?$$

$$A = \frac{9}{B}$$

$$\frac{dB}{dt} = \frac{1}{5}$$

$$\frac{dA}{dt} = -\frac{9}{B^2} \times \frac{dB}{dt}$$

when $A = 12$; $B = \dfrac{9}{12}$

$$= -9 \times \frac{144}{81} \times \frac{1}{5}$$

$$= -\frac{16}{5}$$

$\therefore$ A is decreasing at a rate of 3.2 ms^{-1}

Exercise 9E; 2, 5, 6, 7, 8, 11, 12, 13