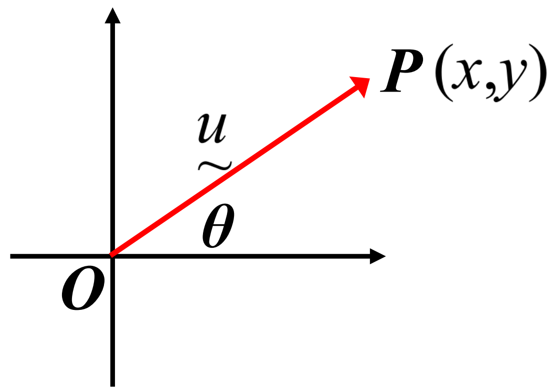
Position Vectors



$\underset{\sim}{u}$ =	$\overrightarrow{OP}$ =	(x,y) =	$\begin{pmatrix} x \\ y \end{pmatrix}$ =	$x\underset{\sim}{i} + y\underset{\sim}{j}$
	position vector	***ordered pair***	***column vector***	***component form***

$$|\underset{\sim}{u}| = \sqrt{x^2 + y^2}$$

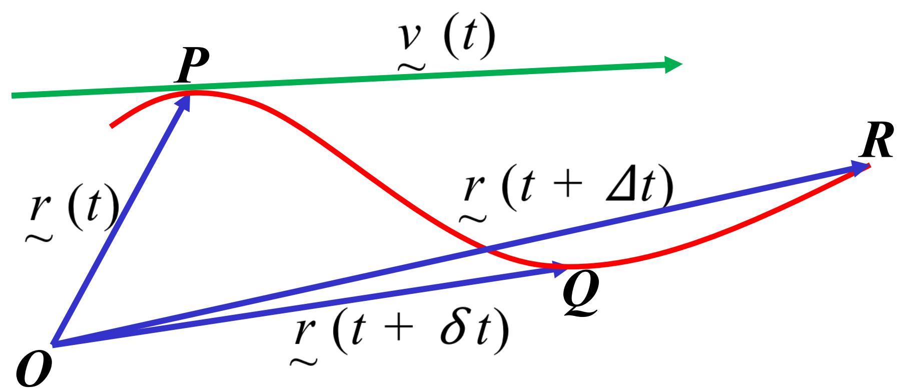
$$\text{direction} = \theta = \tan^{-1}\frac{y}{x}$$

Space Curves

So far all of our motion has been motion in a straight line

Consider a **position vector**, $\underset{\sim}{r}(t)$, as a function of a scalar, t

As t varies $\underset{\sim}{r}$ describes a **space curve**



The **velocity vector,** $\underset{\sim}{v}(t)$, is tangential to the space curve at *P.*

To find velocity and acceleration, and their components, for objects moving in curvilinear paths, the use of vector analysis is needed.

displacement $\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j}$
velocity $\underset{\sim}{v}(t) = \dot{x}(t)\underset{\sim}{i} + \dot{y}(t)\underset{\sim}{j}$ $\lvert\underset{\sim}{v}(t)\rvert = \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}$ direction $= \tan^{-1}\left(\frac{\dot{y}}{\dot{x}}\right)$
acceleration $\underset{\sim}{a}(t) = \ddot{x}(t)\underset{\sim}{i} + \ddot{y}(t)\underset{\sim}{j}$ $\lvert\underset{\sim}{a}(t)\rvert = \sqrt{\ddot{x}(t)^2 + \ddot{y}(t)^2}$

e.g. (i) A particle moves so that its velocity at time t is given by

$$\underset{\sim}{v}(t) = -2\sin 2t\underset{\sim}{i} + 4\cos 2t\underset{\sim}{j} \quad \text{for } 0 \leq t \leq \frac{\pi}{2}$$

Given that $\underset{\sim}{r}(0) = \underset{\sim}{i}$, find the position vector of the particle at any time t.

$$\frac{dx}{dt} = -2\sin 2t$$

$$\int_1^x dx = -2\int_0^t \sin 2t\,dt$$

$$\Big[x\Big]_1^x = \Big[\cos 2t\Big]_0^t$$

$$x - 1 = \cos 2t - 1$$

$$x = \cos 2t$$

$$\frac{dy}{dt} = 4\cos 2t$$

$$\int_0^y dy = 4\int_0^t \cos 2t$$

$$y = 2\Big[\sin 2t\Big]_0^t$$

$$y = 2\sin 2t$$

$$\underset{\sim}{r}(t) = \cos 2t\underset{\sim}{i} + 2\sin 2t\underset{\sim}{j}$$

(ii) The position vectors, at time t, of particles A and $B$are given by;

$$\underset{\sim}{r}_A(t) = (t^3 - 9t + 8)\underset{\sim}{i} + t^2\underset{\sim}{j}$$

$$\underset{\sim}{r}_B(t) = (2 - t^2)\underset{\sim}{i} + (3t - 2)\underset{\sim}{j}$$

Prove that A and B collide while travelling at the same speed, but at right angles to each other.

Particles will collide when they are at the same position at the same time

$$t^3 - 9t + 8 = 2 - t^2$$

$$t^3 + t^2 - 9t + 6 = 0$$

$$t^2 = 3t - 2$$

$$t^2 - 3t + 2 = 0$$

$$(t - 2)(t - 1) = 0$$

$$t = 1 \text{ or } t = 2$$

when $t = 1$, $t^3 + t^2 - 9t + 6 = -1$

when $t = 2$, $t^3 + t^2 - 9t + 6 = 0$

$\therefore$ the two particles collide after 2 seconds

$$\underset{\sim}{r}_A = (t^3 - 9t + 8)\underset{\sim}{i} + t^2\underset{\sim}{j} \qquad \underset{\sim}{r}_B = (2 - t^2)\underset{\sim}{i} + (3t - 2)\underset{\sim}{j}$$

$$\dot{\underset{\sim}{r}}_A = (3t^2 - 9)\underset{\sim}{i} + 2t\underset{\sim}{j} \qquad \dot{\underset{\sim}{r}}_B = -2t\underset{\sim}{i} + 3\underset{\sim}{j}$$

when $t = 2$; $\dot{\underset{\sim}{r}}_A = 3\underset{\sim}{i} + 4\underset{\sim}{j}$ $\qquad \dot{\underset{\sim}{r}}_B = -4\underset{\sim}{i} + 3\underset{\sim}{j}$

$$\left|\dot{\underset{\sim}{r}}_A\right| = \sqrt{3^2 + 4^2} = 5 \qquad \left|\dot{\underset{\sim}{r}}_B\right| = \sqrt{(-4)^2 + 3^2} = 5$$

$\therefore$ when the particles collide they are both travelling at 5 m/s

$$\dot{\underset{\sim}{r}}_A \cdot \dot{\underset{\sim}{r}}_B = \left(3\underset{\sim}{i} + 4\underset{\sim}{j}\right)\cdot\left(-4\underset{\sim}{i} + 3\underset{\sim}{j}\right)$$

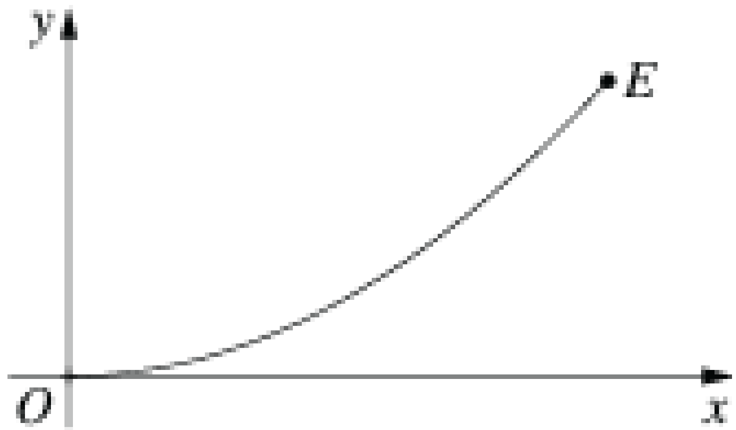
$$= (3)(-4) + (4)(3)$$

$$= 0$$

$\therefore$ the particles collide at right angles to each other

(iii) **2023 Extension 2 HSC Question 9**

A particle travels along a curve from O to E in the xy-plane, as shown in the diagram.



The position vector of the particle is $\boldsymbol{r}$, its velocity is $\boldsymbol{v}$, and its acceleration is $\boldsymbol{a}$.

While travelling from O to E, the particle is always slowing down.

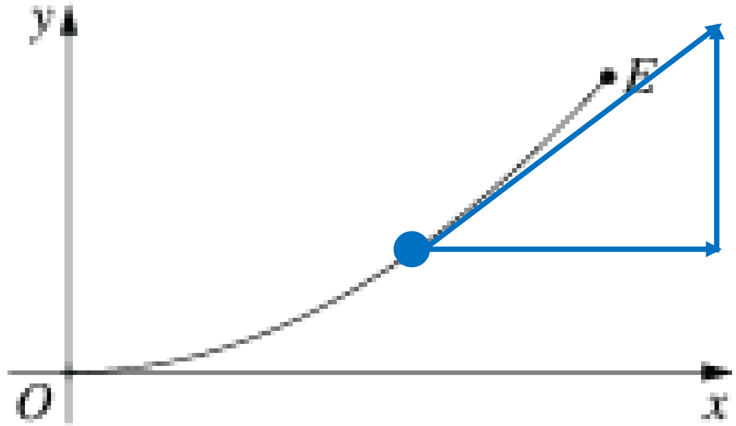
Which of the following is consistent with the motion of the particle?

(A) $\boldsymbol{r}\cdot\boldsymbol{v}\leq 0$ and $\boldsymbol{a}\cdot\boldsymbol{v}\geq 0$

(B) $\boldsymbol{r}\cdot\boldsymbol{v}\leq 0$ and $\boldsymbol{a}\cdot\boldsymbol{v}\leq 0$

(C) $\boldsymbol{r}\cdot\boldsymbol{v}\geq 0$ and $\boldsymbol{a}\cdot\boldsymbol{v}\geq 0$

(D) $\boldsymbol{r}\cdot\boldsymbol{v}\geq 0$ and $\boldsymbol{a}\cdot\boldsymbol{v}\leq 0$



$$\boldsymbol{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} > 0 \\ > 0 \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \geq 0 \\ \geq 0 \end{pmatrix}$$

$$\boldsymbol{r} \cdot \boldsymbol{v} = (> 0)(> 0) + (\geq 0)(\geq 0)$$

$$\geq 0$$

Particle is slowing down

$\therefore$ acceleration is in the opposite direction to velocity

$$\boldsymbol{a} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} \leq 0 \\ \leq 0 \end{pmatrix}$$

$$\boldsymbol{a} \cdot \boldsymbol{v} = (\geq 0)(\leq 0) + (\geq 0)(\leq 0)$$

$$\leq 0$$

$\boldsymbol{r} \cdot \boldsymbol{v} \geq 0$ and $\boldsymbol{a} \cdot \boldsymbol{v} \leq 0$