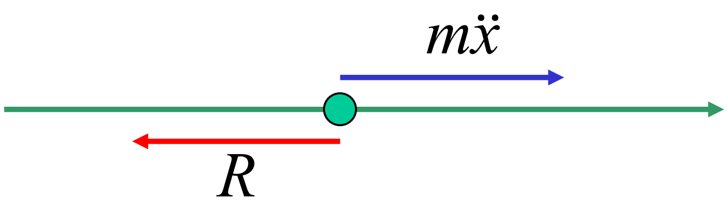
Resisted Motion

Resistance is ALWAYS in the OPPOSITE direction to the motion.

(Newton's 3rd Law)

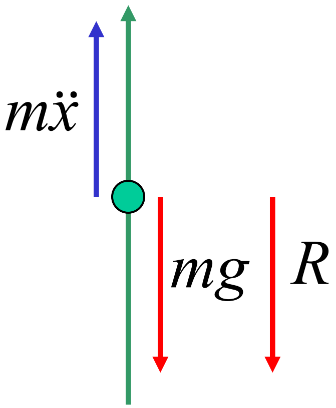
Case 1 (horizontal line)



$$m\ddot{x} = -R$$

$$\ddot{x} = -\frac{R}{m}$$

Case 2 (upwards motion)

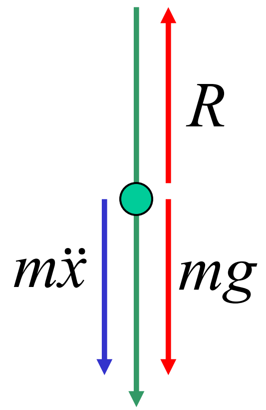


$$m\ddot{x} = -mg - R$$

$$\ddot{x} = -g - \frac{R}{m}$$

NOTE: greatest height still occurs when $v = 0$

Case 3 (downwards motion)

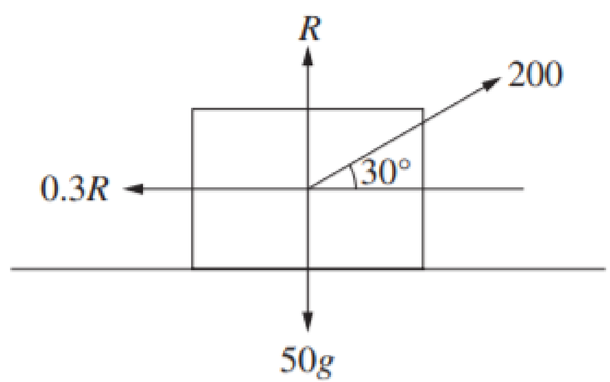


$$m\ddot{x} = mg - R$$

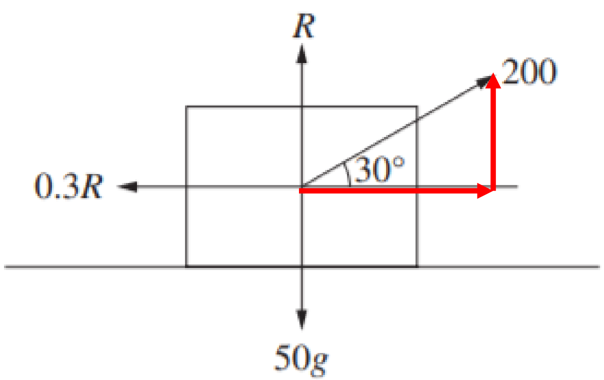
$$\ddot{x} = g - \frac{R}{m}$$

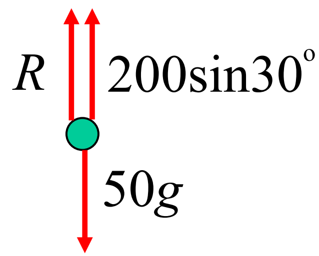
NOTE: terminal velocity occurs when $\ddot{x} = 0$

e.g. (*i*) 2020 Extension 2 HSC Question 12 a)
A 50 kilogram box is initially at rest. The box is pulled along the ground with a force of 200 newtons at an angle of 30° to the horizontal. The box experiences a resistive force of $0.3R$ newtons, where R is the normal force, as shown in the diagram.
Take the acceleration g due to gravity to be 10 m/s^2.



a) By resolving the forces vertically, show that $R = 400$





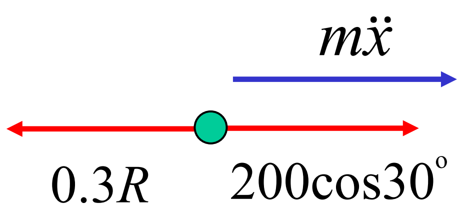
$$\text{vertical } F = 0$$

$$R + 200\sin 30° - 50g = 0$$

$$R + 100 - 500 = 0$$

$$\underline{R = 400}$$

b) Show that the net force horizontally is approximately 53.2 newtons.



net force is equivalent to resultant force

$$m\ddot{x} = 200\cos 30° - 0.3R$$

$$F_H = 100\sqrt{3} - 120$$

$$\underline{= 53.2 \text{ newtons (to 1 dp)}}$$

c) Find the velocity of the box after the first three seconds.

$$m\ddot{x} = 53.2$$

$$\frac{dv}{dt} = \frac{53.2}{50}$$

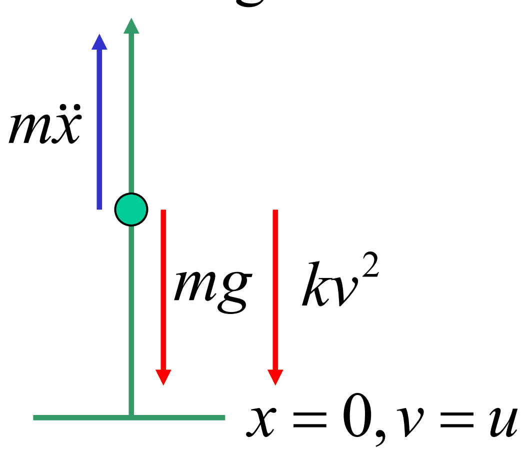
$$= \frac{133}{125}$$

$$\int_0^v dv = \frac{133}{125}\int_0^3 dt$$

$$v = \frac{133}{125}(3)$$

$$= 3.2 \text{ m/s (to 1 dp)}$$

(*ii*) A particle is projected vertically upwards with a velocity of u m/s in a resisting medium.
Assuming that the retardation due to this resistance is equal to kv^2 find expressions for the greatest height reached and the time taken to reach that height.



$$m\ddot{x} = -mg - kv^2$$

$$\ddot{x} = -g - \frac{k}{m}v^2$$

$$v\frac{dv}{dx} = \frac{-mg - kv^2}{m}$$

$$\frac{dx}{dv} = \frac{mv}{-mg - kv^2}$$

$$\int_0^x dx = -m\int_u^o \frac{vdv}{mg + kv^2}$$

$$x = \frac{m}{2k}\int_0^u \frac{2kvdv}{mg + kv^2}$$

$$= \frac{m}{2k}\left[\log\left(mg + kv^2\right)\right]_0^u$$

$$= \frac{m}{2k}\left\{\log\left(mg + ku^2\right) - \log(mg)\right\}$$

$$= \frac{m}{2k}\log\left(\frac{mg + ku^2}{mg}\right)$$

$$= \frac{m}{2k}\log\left(1 + \frac{ku^2}{mg}\right)$$

$\therefore$ the greatest height

is $\frac{m}{2k}\log\left(1 + \frac{ku^2}{mg}\right)$ metres

$$\ddot{x} = -g - \frac{k}{m}v^2$$

$$\frac{dv}{dt} = \frac{-mg - kv^2}{m}$$

$$\int_0^t dt = -m\int_u^0 \frac{dv}{mg + kv^2}$$

$$t = \frac{m}{k}\int_0^u \frac{dv}{\frac{mg}{k} + v^2}$$

$$= \frac{m}{k}\left[\sqrt{\frac{k}{mg}}\tan^{-1}\left(\sqrt{\frac{k}{mg}}v\right)\right]_0^u$$

$$= \sqrt{\frac{m}{kg}}\left\{\tan^{-1}\left(\sqrt{\frac{k}{mg}}u\right) - \tan^{-1}0\right\}$$

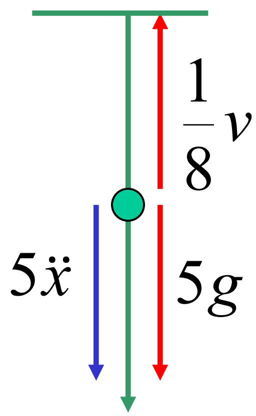
$$= \sqrt{\frac{m}{kg}}\tan^{-1}\left(\sqrt{\frac{k}{mg}}u\right)$$

$\therefore$ it takes $\sqrt{\frac{m}{kg}}\tan^{-1}\left(\sqrt{\frac{k}{mg}}u\right)$ seconds to reach the greatest height.

(iii) A body of mass 5kg is dropped from a height at which the gravitational acceleration is g.

Assuming that air resistance is proportional to speed v, the constant of proportion being $\frac{1}{8}$, find;

a) the velocity after time t.



$$5\ddot{x} = 5g - \frac{1}{8}v$$

$$\ddot{x} = g - \frac{1}{40}v$$

$$\frac{dv}{dt} = \frac{40g - v}{40}$$

$$\int_0^t dt = 40\int_0^v \frac{dv}{40g - v}$$

$$t = -40\left[\log(40g - v)\right]_0^v$$

$$= -40\left\{\log(40g - v) - \log(40g)\right\}$$

$$= 40\log\left(\frac{40g}{40g - v}\right)$$

$$\frac{t}{40}=\log\left(\frac{40g}{40g-v}\right)$$

$$\frac{40g}{40g-v}=e^{\frac{t}{40}}$$

$$\frac{40g-v}{40g}=e^{-\frac{t}{40}}$$

$$40g-v=40ge^{-\frac{t}{40}}$$

$$v=40g-40ge^{-\frac{t}{40}}$$

$$v=40g\left(1-e^{-\frac{t}{40}}\right)$$

b) the terminal velocity

terminal velocity occurs when $\ddot{x}=0$

$$\text{i.e.} 0=g-\frac{1}{40}v$$

$$v=40g$$

OR

$$\lim_{t\to\infty}v=\lim_{t\to\infty}40g\left(1-e^{-\frac{t}{40}}\right)$$

$$=40g$$

$\therefore$ terminal velocity is $40g$ m/s

c) The distance it has fallen after time t

$$\frac{dx}{dt} = 40g\left(1 - e^{-\frac{t}{40}}\right)$$

$$\int_0^x dx = 40g\int_0^t \left(1 - e^{-\frac{t}{40}}\right) dt$$

$$x = 40g\left[t + 40e^{-\frac{t}{40}}\right]_0^t$$

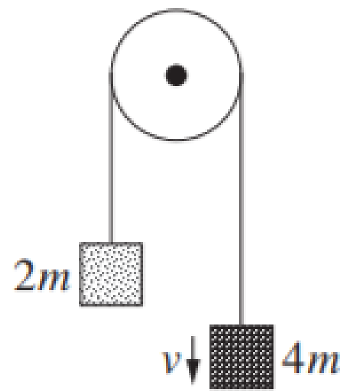
$$x = 40g\left\{t + 40e^{-\frac{t}{40}} - 0 - 40\right\}$$

$$\underline{x = 40gt + 1600ge^{-\frac{t}{40}} - 1600g}$$

(*iv*) 2020 Extension 2 HSC Question 16 a)

Two masses, $2m$ kg and $4m$ kg, are attached by a light string. The string is placed over a smooth pulley as shown.

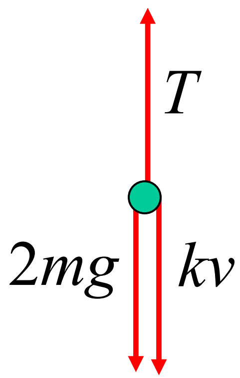
The two masses are at rest before being released and v is the velocity of the larger mass at time t seconds after they are released.



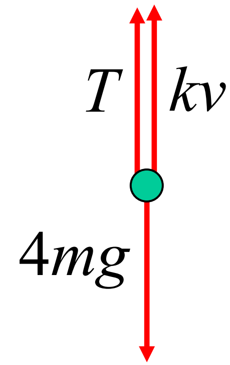
The force due to air resistance on each mass has magnitude kv, where k is a positive constant.

a) Show that $\dfrac{dv}{dt} = \dfrac{gm - kv}{3m}$

Forces on left mass



Forces on right mass



$$2m\ddot{x} = T - 2mg - kv$$
$$T = 2m\ddot{x} + 2mg + kv$$

$$4m\ddot{x} = 4mg - T - kv$$
$$= 4mg - (2m\ddot{x} + 2mg + kv) - kv$$
$$6m\ddot{x} = 2mg - 2kv$$
$$\ddot{x} = \frac{mg - kv}{3m}$$
$$\frac{dv}{dt} = \frac{gm - kv}{3m}$$

b) Given that $v < \dfrac{gm}{k}$, show that when $t = \dfrac{3m}{k}\ln 2$, the velocity of the larger mass is $\dfrac{gm}{2k}$

$$\frac{dv}{dt} = \frac{mg - kv}{3m}$$

$$\int_0^v \frac{3m\,dv}{mg - kv} = \int_0^{\frac{3m}{k}\ln 2} dt$$

$$-\frac{3m}{k}\int_0^v \frac{-k\,dv}{mg - kv} = \frac{3m}{k}\ln 2$$

$$\left[\ln|mg - kv|\,\right]_v^0 = \ln 2$$

$$\ln\left|\frac{mg}{mg - kv}\right| = \ln 2$$

as $v < \dfrac{gm}{k} \Rightarrow \dfrac{mg}{mg - kv} > 0$

$$\frac{mg}{mg - kv} = 2$$

$$mg = 2mg - 2kv$$

$$2kv = mg$$

$$v = \frac{mg}{2k}$$

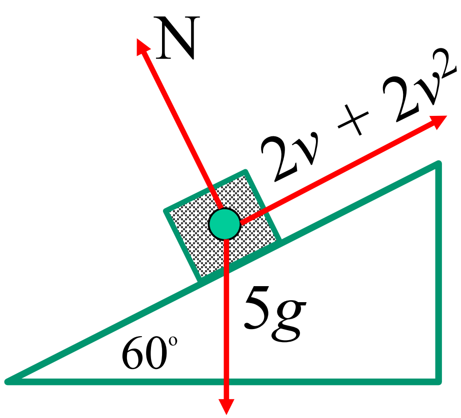
(v) 2021 Extension 2 HSC Question 14 b)

An object of mass 5 kg is on a slope that is inclined at an angle of 60° to the horizontal. The acceleration due to gravity is g ms^{-2} and the velocity of the object down the slope is v ms^{-1}.

As well as the force due to gravity, the object is acted upon by two forces, one of magnitude $2v$ newtons and one of magnitude $2v^2$ newtons, both acting up the slope.

a) Show that the resultant force down the slope is

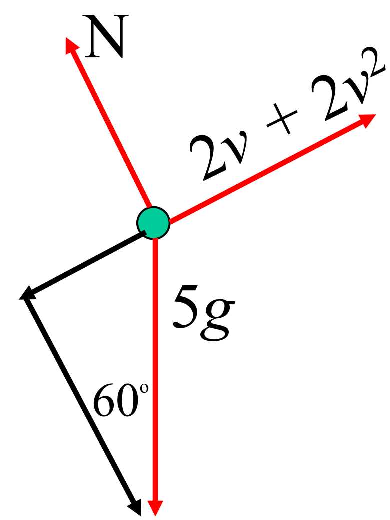
$$\frac{5\sqrt{3}}{2}g - 2v - 2v^2 \text{ newtons.}$$



resolving forces down the plane

$$m\ddot{x} = 5g\sin 60^\circ - 2v - 2v^2$$

$$= \frac{5\sqrt{3}}{2}g - 2v - 2v^2$$



b) There is one value of v such that the object will slide down the slope at a constant speed.

Find this value of v in ms^{-1}, correct to 1 decimal place, given that $g = 10$.

If the object slides down the slope at a constant speed, $\ddot{x} = 0$.

$$\text{i.e. } \frac{5\sqrt{3}}{2}g - 2v - 2v^2 = 0$$

$$25\sqrt{3} - 2v - 2v^2 = 0$$

$$v = \frac{2 \pm \sqrt{4 + 200\sqrt{3}}}{-4}$$

$$\text{but } v > 0 \text{ ; } v = \frac{2 - \sqrt{4 + 200\sqrt{3}}}{-4}$$

$$= 4.2 \text{ ms}^{-1} \quad \text{(to 1 dp)}$$

Exercise 6D;
1, 2, 4, 5, 6, 8, 10, 12

Exercise 6E;
1 to 8, 10, 13