Sequences & Series

Definitions

Sequence (Progression): a set of numbers that follow a pattern

Series: a set of numbers added together

a: the first term

T_n : the nth term

S_n : the sum of the first n terms

e.g. $T_n = n^2 + 2$, find;

(i) $T_5 = 5^2 + 2$

$= 27$

(ii) whether 42 is a term in the sequence

$42 = n^2 + 2$

$n^2 = 40$

$n = \sqrt{40}$, which is not an integer

Thus 42 is not a term

Recursive Formulae: defines the series in terms of previous terms

A recursive formula designates the;

* starting term or terms
* an expression for the general term containing the previous term(s)

e.g. (i) Write down the first four terms of the sequence defined by

$$T_n = T_{n-1} + 3 \text{ for } n \geq 2, T_1 = 4$$

$T_1 = 4$

$T_2 = 4 + 3 = 7$

$T_3 = 7 + 3 = 10$

$T_4 = 10 + 3 = 13$

(ii) The Lucas numbers are defined as;

$$\boldsymbol{L}_n = \boldsymbol{L}_{n-1} + \boldsymbol{L}_{n-2} \text{ for } n \geq 3 \ , \ \boldsymbol{L}_1 = 2 \ , \ \boldsymbol{L}_2 = 1$$

find the first five Lucas numbers

$$2 \ , 1 \ , 3 \ , 4 \ , 7$$

Arithmetic Series

An arithmetic series is a sequence of numbers in which each term after the first is found by **adding** a constant amount to the previous term.

The constant amount is called the **common difference**, symbolised, ***d***.

$$d = T_2 - a$$
$$= T_3 - T_2$$
$$d = T_n - T_{n-1}$$

$$T_n = T_{n-1} + d \quad \textbf{\textit{(recursive formula)}}$$

$$T_1 = a$$
$$T_2 = a + d$$
$$T_3 = a + 2d$$
$$T_n = a + (n-1)d$$

$e.g.(i)$ If $T_3 = 9$ and $T_7 = 21$, find;

the general term.

$$a + 2d = 9$$
$$a + 6d = 21$$
$$4d = 12$$
$$d = 3 \ \therefore a = 3$$

$$T_n = 3 + (n-1)3$$
$$= 3 + 3n - 3$$
$$= 3n$$

$(ii)\ T_{100} = 3(100)$

$= 300$

(iii) the first term greater than 500

$T_n > 500$

$3n > 500$

$n > \dfrac{500}{3}$

$\therefore n = 167$

$T_{167} = 501$, is the first term > 500

When plotted on a number plane, the graph of an arithmetic sequence is a linear function

Exercise 1A; 2aceg, 3bdf, 5acdf, 7bef, 12ac, 13bd, 14c, 16, 17aceg, 18, 19, 20

Exercise 1B; 7adg, 9c, 10be, 11, 13ac, 16, 17b, 18b, 19acf, 20