Series – The Sum Of A Sequence

$$T_1 + T_2 + T_3 + \ldots + T_n = S_n = \sum_{k=1}^{n} T_k$$

$$e.g.\ (i) \sum_{n=1}^{5} 2n+3 = (2+3)+(4+3)+(6+3)+(8+3)+(10+3)$$
$$= 45$$

(ii) If $S_{10} = 29$ and $S_{11} = 37$, find T_{11}

$$T_{11} = S_{11} - S_{10}$$
$$= 37 - 29$$
$$= 8$$

(iii) If $S_n = 5n^2 - 2$, find T_n

$$
\begin{aligned}
T_n &= S_n - S_{n-1} \\
&= 5n^2 - 2 - [5(n-1)^2 - 2] \\
&= 5n^2 - 2 - 5n^2 + 10n - 5 + 2 \\
&= 10n - 5
\end{aligned}
$$

Sum Of An Arithmetic Series

$$S_n = a + a + d + a + 2d + \ldots + l - 2d + l - d + l$$
$$S_n = l + l - d + l - 2d + \ldots + a + 2d + a + d + a$$

$$2S_n = a + l + a + l + a + l + \ldots + a + l + a + l + a + l$$
$$= n(a + l)$$

$$\boxed{S_n = \frac{n}{2}(a + l)}$$ if we know l

otherwise;

$$S_n = \frac{n}{2}\{a + a + (n-1)d\}$$

$$\boxed{S_n = \frac{n}{2}\{2a + (n-1)d\}}$$

$e.g.(i)$ If $a = 3$ and $T_6 = 96$, find S_6

$$S_n = \frac{n}{2}(a + l)$$

$$S_6 = \frac{6}{2}(3 + 96)$$

$$= 297$$

(ii) Find the sum of the first 100 even numbers

$$a = 2, d = 2 \text{ and } n = 100$$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_{100} = \frac{100}{2}\{4 + (99)(2)\}$$

$$= 50 \times 202$$

$$= 10100$$

(iii) The sum of the first 10 numbers is 100 and the first 5 numbers is 25.

Find a, d and the general term.

$$S_{10} = 100$$
$$\frac{10}{2}\{2a + 9d\} = 100$$
$$2a + 9d = 20$$

$$S_5 = 25$$
$$\frac{5}{2}\{2a + 4d\} = 25$$
$$a + 2d = 5$$

$$2a + 9d = 20$$
$$2a + 4d = 10$$
$$5d = 10$$
$$d = 2 \quad \therefore a = 1$$

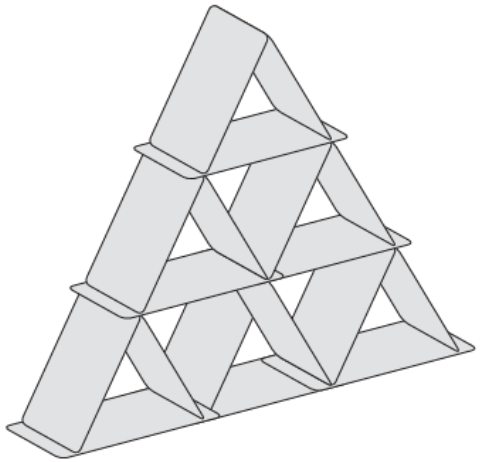
$$T_n = a + (n-1)d$$
$$= 1 + (n-1)2$$
$$= 2n - 1$$

$$\therefore a = 1, d = 2, T_n = 2n - 1$$

(*iv*) **2022 Advanced HSC Q17**

Cards are stacked to build a 'house of cards'. A house of cards with 3 rows is shown.

A house of cards requires 3 cards in the top row, 6 cards in the next row, and each successive row has 3 more cards than the previous row.



a) Show that a house of cards with 12 rows has a total of 234 cards.

$a = 3$, $d = 3$ and $n = 12$

In a **"show that"** question, don't be lazy, show your substitution.
Make it obvious how you got your answer

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_{12} = \frac{12}{2}\{2(3) + (12-1)(3)\}$$

$$= 6(6 + 33)$$

$$= 234$$

b) Another house of cards has a total of 828 cards.

How many rows are in this house of cards?

$$S_n = 828$$

$$\frac{n}{2}\{6 + (n-1)(3)\} = 828$$

$$\frac{n}{2}(3 + 3n) = 828$$

$$3n + 3n^2 = 1\,656$$

$$n^2 + n - 552 = 0$$

$$(n + 24)(n - 23) = 0$$

$$n = -24 \quad \text{or} \quad n = 23$$

$$\text{however } n > 0$$

$\therefore$ there are 23 rows in the house of cards

$$(v)\ \sum_{n=1}^{10}(3n-6)$$

$$a=-3,\ l=24,\ n=10$$

$$S_n=\frac{n}{2}(a+l)$$

$$S_{10}=\frac{10}{2}(-3+24)$$

$$=105$$

Exercise 1E; 4ace etc, 8, 9b, 10bdf, 11bdf, 12, 13, 15

Exercise 1F; 4ace, 6bd, 7ce, 9c, 10ad, 11a, 13, 14c, 16ac, 17a, 18