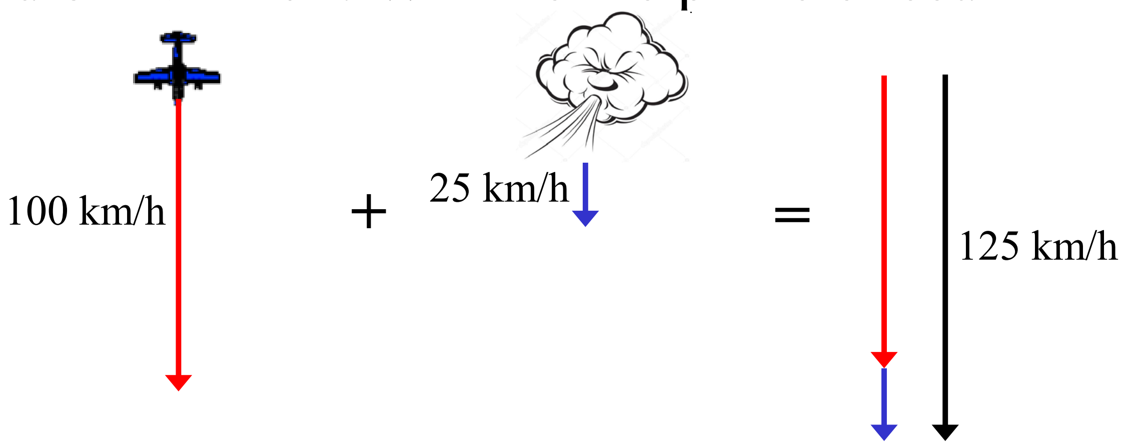
Dynamics & Resolving Vectors

In Dynamics, problems are often solved by looking at what is happening horizontally, and what is happening vertically.

These two directions can then be resolved into one vector describing the overall situation.

an example in one dimension

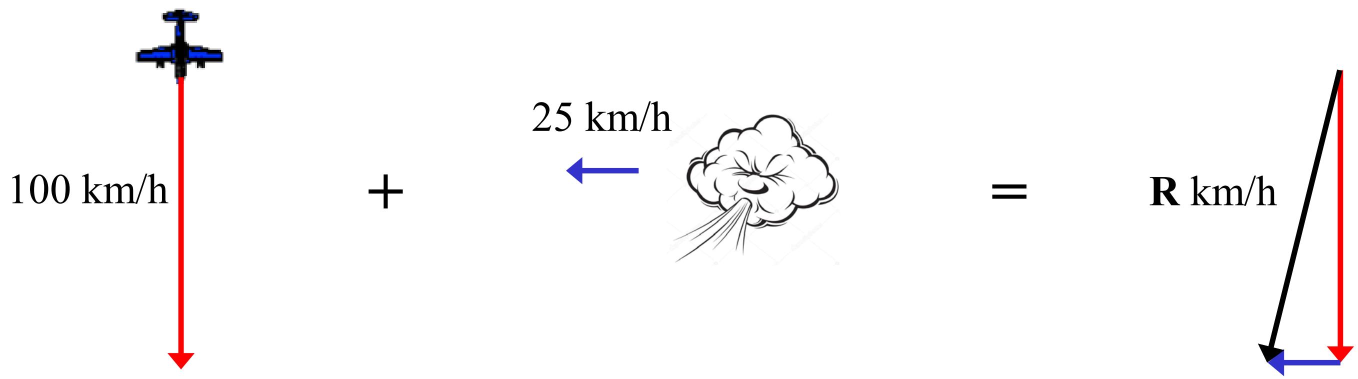
A plane is travelling south with a velocity of 100 km/h and experiences a tailwind of 25 km/h. What is the plane's resultant velocity?



The plane's resultant velocity is 125 km/h

an example in two dimensions

A plane is travelling south with a velocity of 100 km/h and experiences an easterly sidewind of 25 km/h. What is the plane's resultant velocity?



$$R^2 = 100^2 + 25^2$$

$$R = \sqrt{10625}$$

$$= 103.0776...$$

The plane's resultant velocity is 103 km/h

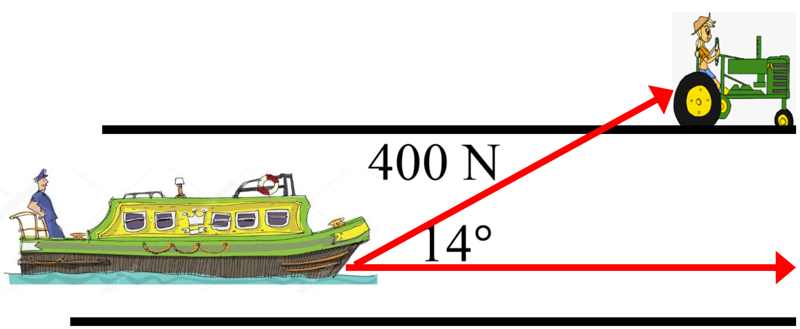
Force is a vector quantity

i.e. it has magnitude and direction

e.g. The forces $F_1 = \left(3\underset{\sim}{i} + 2\underset{\sim}{j}\right)$ N , $F_2 = \left(6\underset{\sim}{i} - 4\underset{\sim}{j}\right)$ N and $F_3 = \left(2\underset{\sim}{i} - \underset{\sim}{j}\right)$ N act on a particle. Find the resultant force on the particle

$$F_R = \left(3\underset{\sim}{i} + 2\underset{\sim}{j}\right) + \left(6\underset{\sim}{i} - 4\underset{\sim}{j}\right) + \left(2\underset{\sim}{i} - \underset{\sim}{j}\right)$$
$$= \underline{\left(11\underset{\sim}{i} - 3\underset{\sim}{j}\right) \text{N}}$$

(ii) A tractor is pulling a barge along a canal with a force of 400 N. The barge is moving parallel to the bank. Find the component of F in the direction of motion.

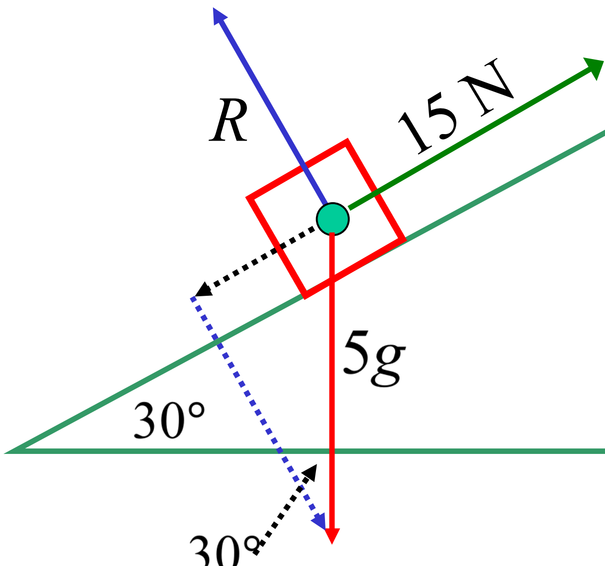


$$\text{proj}_{\underset{\sim}{b}}\, \underset{\sim}{t} = (|\underset{\sim}{t}|\cos\theta)\underset{\sim}{i}$$
$$= 400\cos 14°$$
$$= 388.11829\ldots$$

horizontal component of F = 388 N

(iii) A particle of mass 5 kg lies on a smooth plane inclined at 30° to the horizontal. There is a frictional force of 15 N acting up the plane and a gravitational force (mg) acting downwards.

Find the acceleration of the particle down the incline and the reaction force R.



resolving forces down the plane

$$F_P = 5g\sin 30° - 15$$

$$F_P = (5)(9.8)\left(\frac{1}{2}\right) - 15$$

$$5a_P = 9.5$$

$$a_P = 1.96$$

$\therefore$ resultant acceleration is $1.96\ \text{ms}^{-2}$ down the plane

resolving forces perpendicular to the plane

$$F_{\perp} = R - 5g\cos 30°$$

$$0 = R - 5g\left(\frac{\sqrt{3}}{2}\right)$$

$$R = \frac{5\sqrt{3}}{2}g\ \text{N}$$

Exercise 8F; 1, 3 to 7, 9, 10, 12, 14 to 17, 19