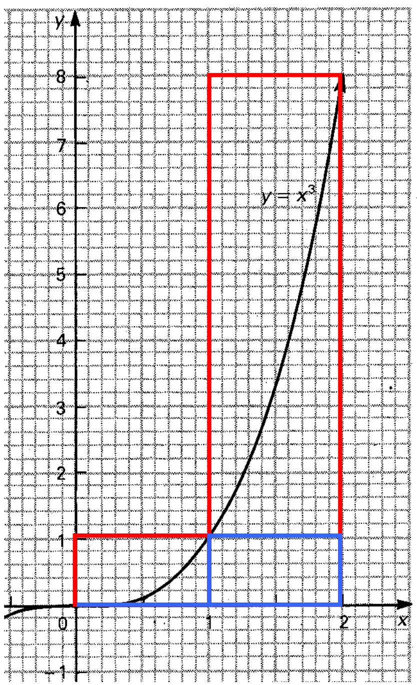
Integration
Area Under Curve

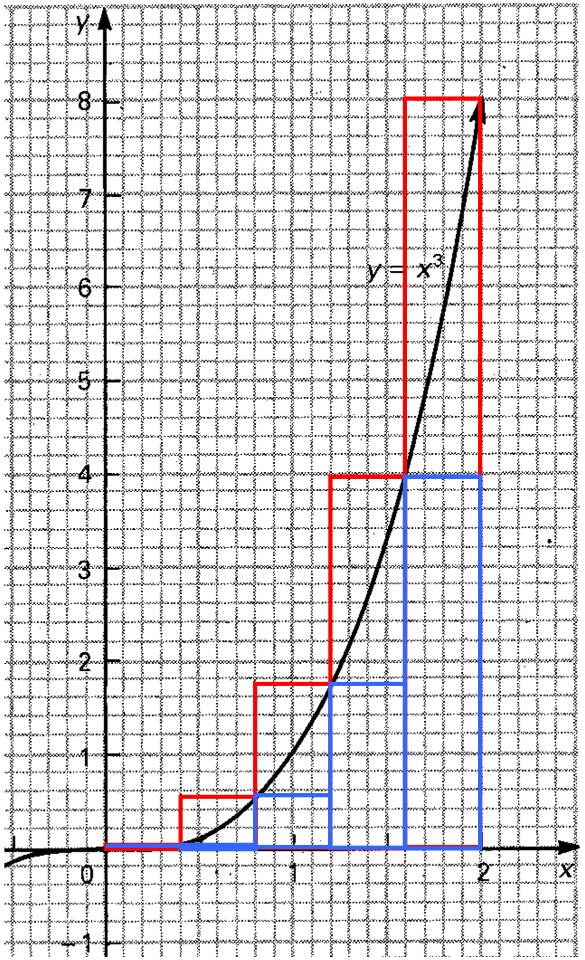


$$1(0)^3 + 1(1)^3 \leq \text{Area} \leq 1(1)^3 + 1(2)^3$$

$$1 \leq \text{Area} \leq 9$$

$$\text{Estimate Area} = 5\ \text{unit}^2$$

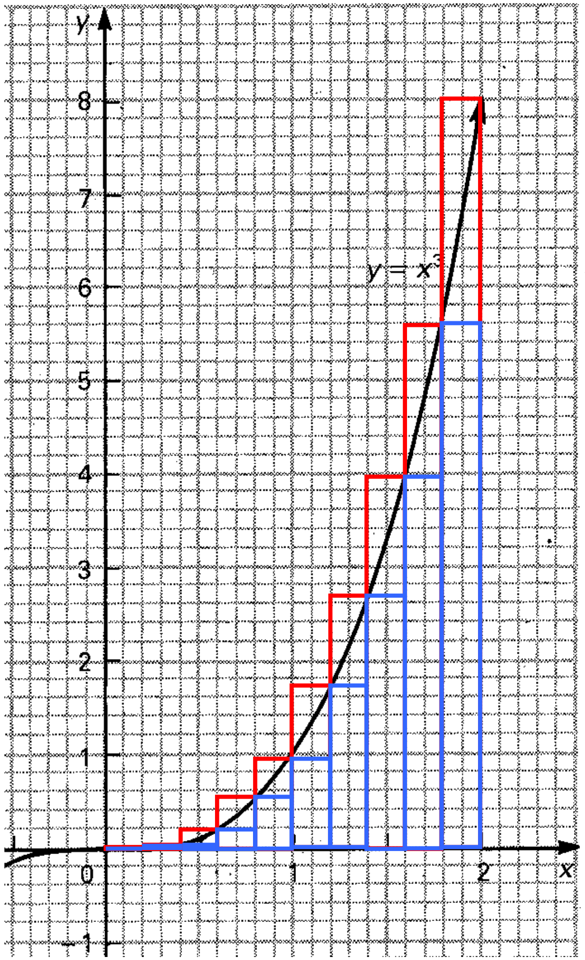
$$\left(Exact\ Area = 4\ unit^2\right)$$



$$0.4\{0^3 + 0.4^3 + 0.8^3 + 1.2^3 + 1.6^3\} \le \text{Area} \le 0.4\{0.4^3 + 0.8^3 + 1.2^3 + 1.6^3 + 2^3\}$$

$$\underline{2.56 \le \text{Area} \le 5.76}$$

$$\text{Estimate Area} = 4.16\ \text{unit}^2$$



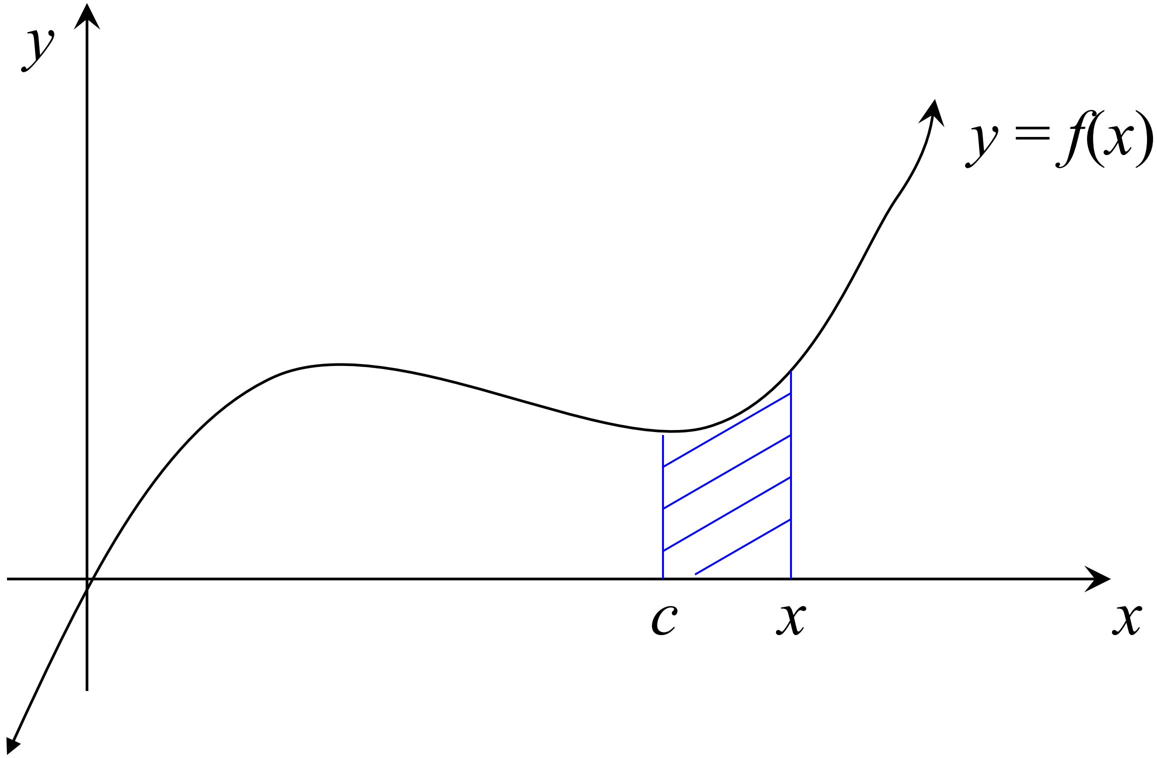
$$0.2\{0^3+0.2^3+0.4^3+0.6^3+0.8^3+1^3+1.2^3+1.4^3+1.6^3+1.8^3\}\le$$

$$\text{Area}\le 0.2\{0.2^3+0.4^3+0.6^3+0.8^3+1^3+1.2^3+1.4^3+1.6^3+1.8^3+2^3\}$$

$$3.24\le \text{Area}\le 4.84$$

$$\text{Estimate Area} = 4.04\ \text{unit}^2$$

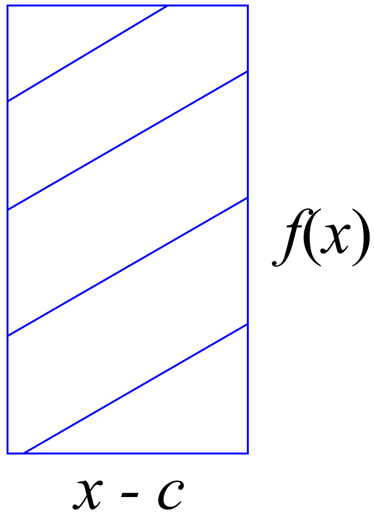
As the widths decrease, the estimate becomes more accurate, lets investigate **one** of these rectangles.



$A(c)$ is the area from 0 to c

$A(x)$ is the area from 0 to x

$\therefore$ **$A(x) - A(c)$** denotes the area from c to x, and can be estimated by the rectangle;



$$A(x) - A(c) \approx (x-c)f(x)$$

$$f(x) \approx \frac{A(x)-A(c)}{x-c}$$

$$= \frac{A(c+h)-A(c)}{h} \qquad h = \text{width of rectangle}$$

As the width of the rectangle decreases, the estimate becomes more accurate.

i.e. as $h \to 0$, the Area becomes exact

$$f(x) = \lim_{h \to 0} \frac{A(c+h) - A(c)}{h}$$

$$= \lim_{h \to 0} \frac{A(x+h) - A(x)}{h} \qquad (\because \text{as } h \to 0, c \to x)$$

$$= A'(x)$$

$\therefore$ the equation of the curve is the derivative of the Area function.

The area under the curve $y = f(x)$ between $x = a$ and $x = b$ is;

$$A = \int_a^b f(x)dx$$

$$= F(b) - F(a)$$

where $F(x)$ is the primitive function of $f(x)$

e.g. (i) Find the area under the curve $y = x^3$, between $x = 0$ and $x = 2$

$$A = \int_0^2 x^3 dx$$

$$= \left[\frac{1}{4}x^4\right]_0^2$$

$$= \frac{1}{4}\left\{2^4 - 0^4\right\}$$

$$= 4 \text{ units}^2$$

$$(ii) \int_2^3 \left(x^2 + 1\right) dx = \left[\frac{1}{3}x^3 + x\right]_2^3$$

$$= \left\{\frac{1}{3}(3)^3 + 3\right\} - \left\{\frac{1}{3}(2)^3 + 2\right\}$$

$$= \frac{22}{3}$$

$$(iii) \int_4^5 x^{-3} dx = \left[-\frac{1}{2}x^{-2}\right]_4^5$$

$$= -\frac{1}{2}\left\{\frac{1}{5^2} - \frac{1}{4^2}\right\}$$

$$= \frac{9}{800}$$

$$(iv)\ \frac{d}{dx}\int_c^x f(t)\,dt = \frac{d}{dx}\Big[F(x)\Big]_c^x$$

$$= \frac{d}{dx}\{F(x) - F(c)\}$$

$$= f(x) - 0$$

$$\frac{d}{dx}\int_c^x f(t)\,dt = f(x)$$

Exercise 5A; 1, 2, 3, 4bdg, 5g, 6i, 7a, 8, 9, 11,

Exercise 5B; 1a, 2a ii, 3ah, 5e, 6af, 7bf, 9ab (i, ii), 10b, 11df, 12ab, 13c, 14b, 15, 16a ii, 17