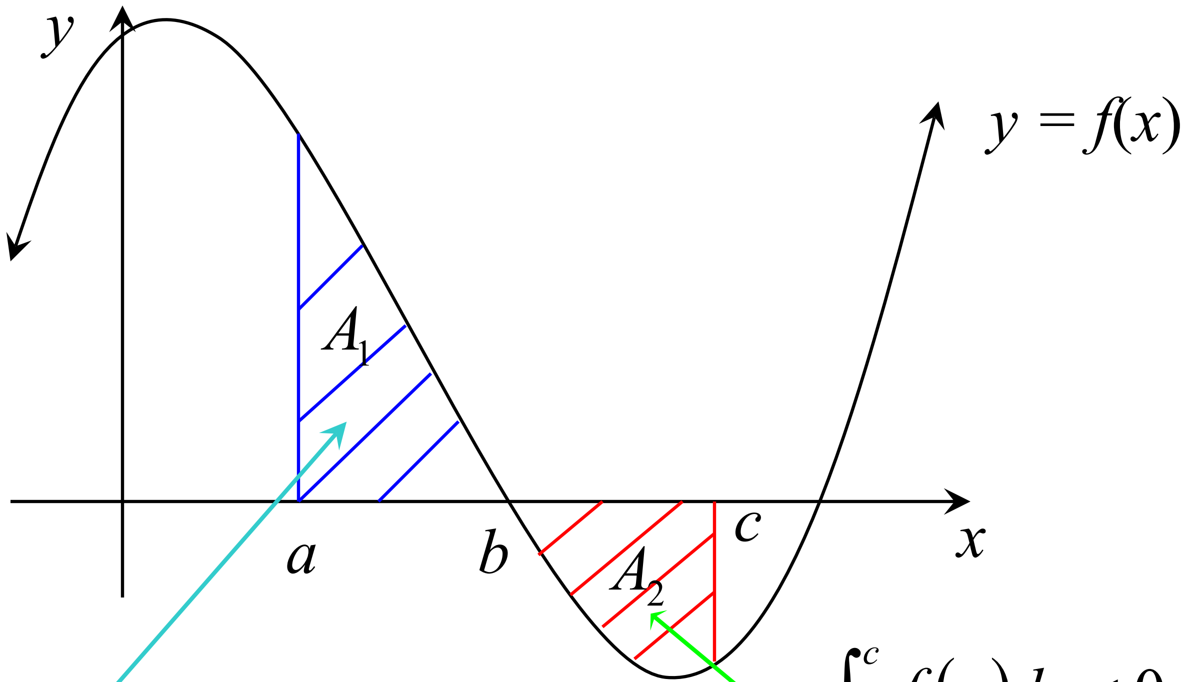
Areas

(1) Area Below *x* axis



$$\int_a^b f(x)dx > 0$$

$$\therefore A_1 = \int_a^b f(x)dx$$

$$\int_b^c f(x)dx < 0$$

$$\therefore A_2 = -\int_b^c f(x)dx$$

Area below x axis is given by;

$$A = -\int_b^c f(x)dx$$

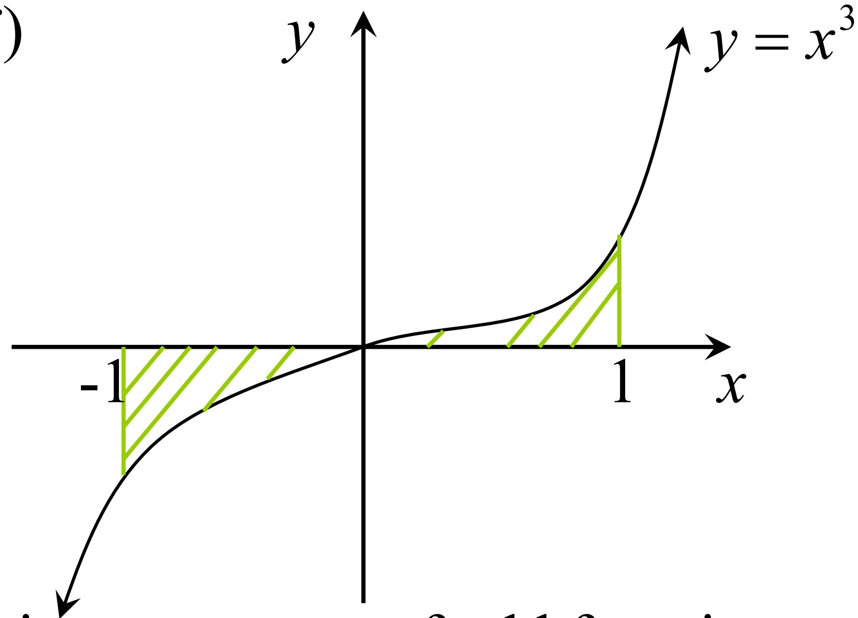
OR

$$= \left|\int_b^c f(x)dx\right|$$

OR

$$= \int_c^b f(x)dx$$

e.g. (i)



$$A = -\int_{-1}^{0} x^3 dx + \int_0^1 x^3 dx$$

$$= -\frac{1}{4}\left[x^4\right]_{-1}^{0} + \frac{1}{4}\left[x^4\right]_0^1$$

$$= -\frac{1}{4}\left\{0 - (-1)^4\right\} + \frac{1}{4}\left\{1^4 - 0\right\}$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2} \text{ units}^2$$

OR using symmetry of odd function

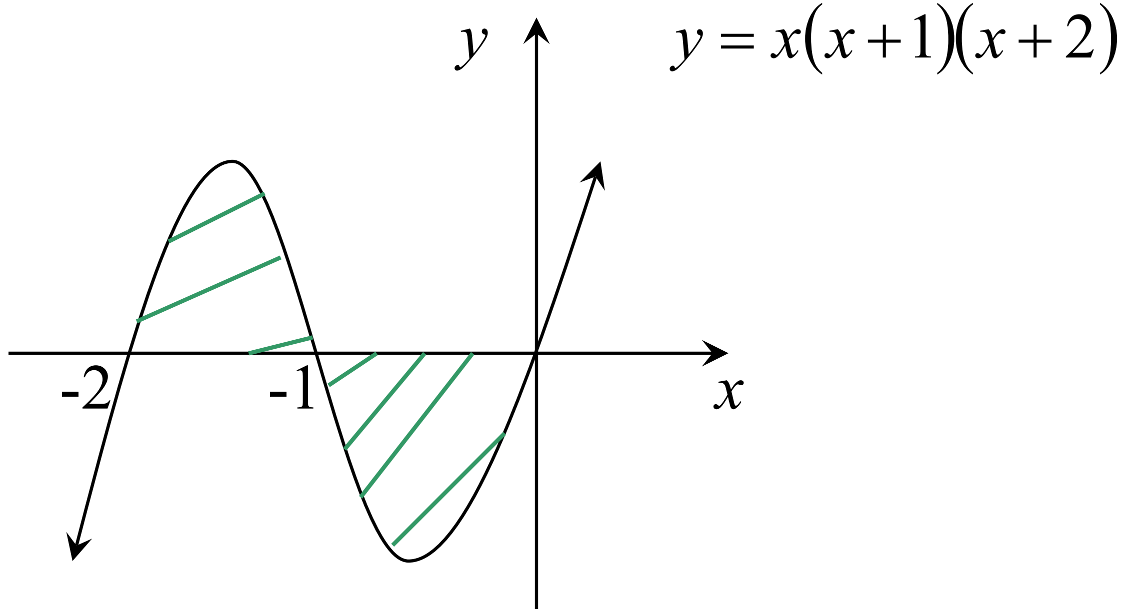
$$A = 2\int_0^1 x^3 dx$$

$$= \frac{1}{2}\left[x^4\right]_0^1$$

$$= \frac{1}{2}\left\{1^4 - 0\right\}$$

$$= \frac{1}{2} \text{ units}^2$$

(*ii*)



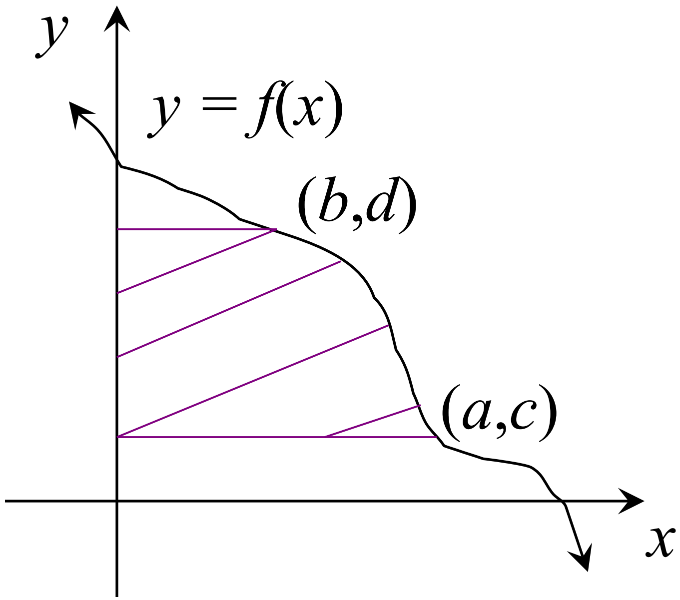
$$A = \int_{-2}^{-1}\left(x^3 + 3x^2 + 2x\right)dx - \int_{-1}^{0}\left(x^3 + 3x^2 + 2x\right)dx$$

$$= \left[\frac{1}{4}x^4 + x^3 + x^2\right]_{-2}^{-1} + \left[\frac{1}{4}x^4 + x^3 + x^2\right]_{0}^{-1}$$

$$= 2\left\{\frac{1}{4}(-1)^4 + (-1)^3 + (-1)^2\right\} - \left\{\frac{1}{4}(-2)^4 + (-2)^3 + (-2)^2\right\} - 0$$

$$= \frac{1}{2}\text{ units}^2$$

(2) **Area On The y axis**



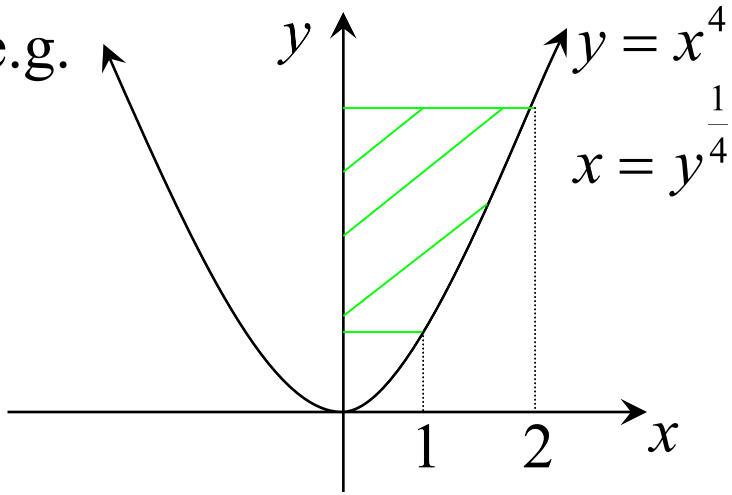
(1) Make x the subject

i.e. $x = g(y)$

(2) Substitute the y coordinates

(3) $$A = \int_c^d g(y)\,dy$$

e.g.



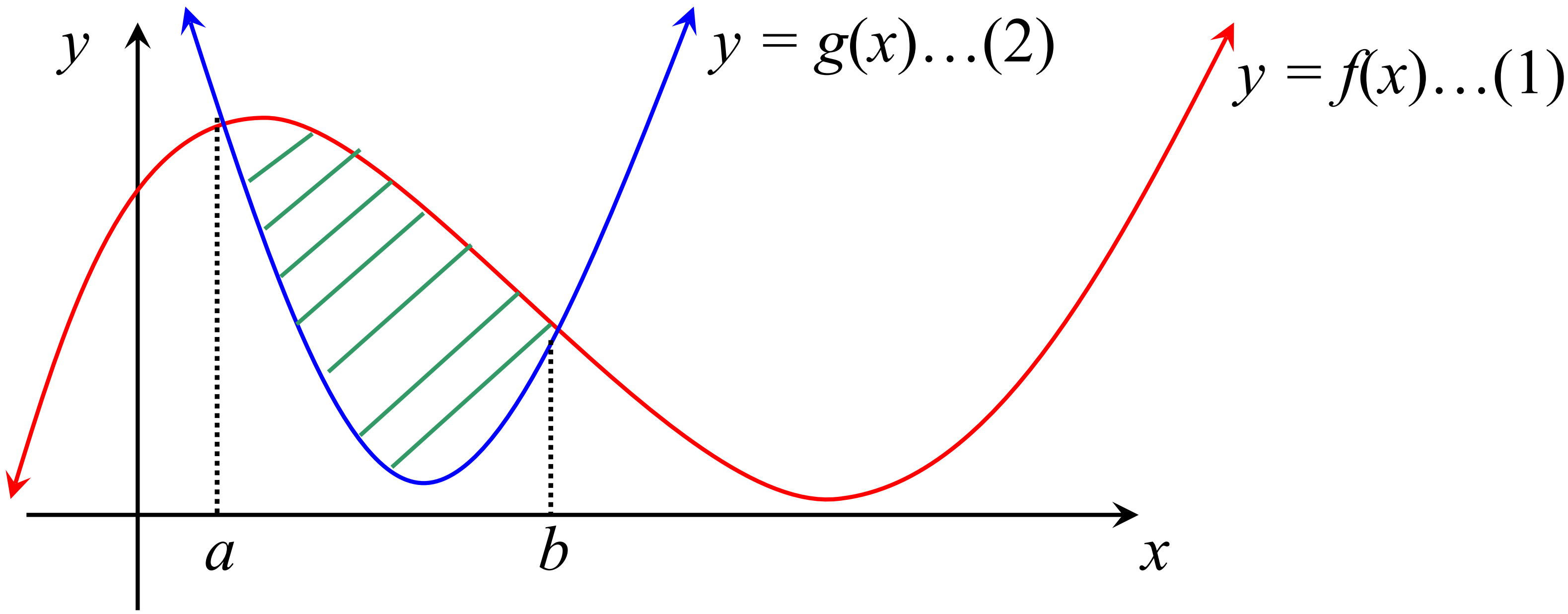
$$A = \int_1^{16} y^{\frac{1}{4}}\,dy$$

$$= \frac{4}{5}\left[y^{\frac{5}{4}} \right]_1^{16}$$

$$= \frac{4}{5}\left\{ 16^{\frac{5}{4}} - 1^{\frac{5}{4}} \right\}$$

$$= \frac{124}{5} \text{ units}^2$$

(3) Area Between Two Curves

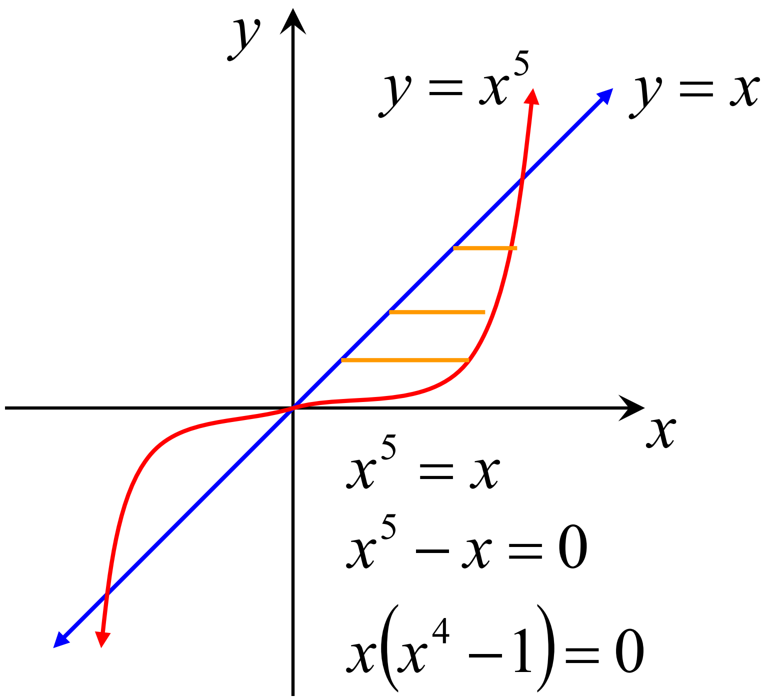


$$\text{Area} = \text{Area under (1)} - \text{Area under (2)}$$

$$= \int_a^b f(x)dx - \int_a^b g(x)dx$$

$$= \int_a^b \left(f(x) - g(x)\right)dx$$

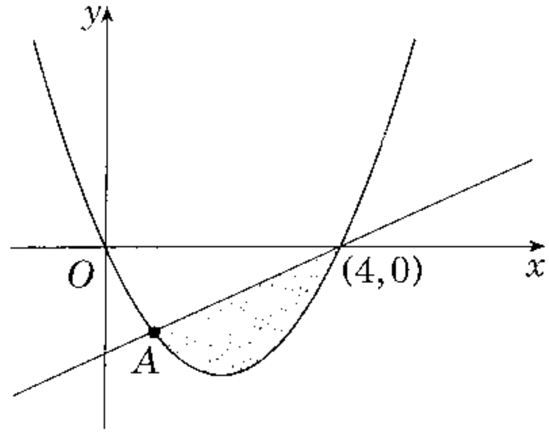
e.g. Find the area enclosed between the curves $y = x^5$ and $y = x$ in the positive quadrant.



$$x^5 = x$$
$$x^5 - x = 0$$
$$x\left(x^4 - 1\right) = 0$$
$$x = 0 \text{ or } x = \pm 1$$

$$A = \int_0^1 \left(x - x^5\right) dx$$
$$= \left[\frac{1}{2}x^2 - \frac{1}{6}x^6\right]_0^1$$
$$= \left\{\frac{1}{2}(1)^2 - \frac{1}{6}(1)^6\right\} - 0$$
$$= \frac{1}{3} \text{ unit}^2$$

2002 HSC Question 4d)



The graphs of $y = x - 4$ and $y = x^2 - 4x$ intersect at the points $(4,0)$ A.

(i) Find the coordinates of A **(2)**

To find points of intersection, solve simultaneously

$$x - 4 = x^2 - 4x$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0$$

$$x = 1 \quad \text{or} \quad x = 4$$

$$\therefore A \text{ is } (1,-3)$$

(ii) Find the area of the shaded region bounded by $y = x^2 - 4x$ and $y = x - 4$. **(3)**

$$A = \int_1^4 \left(x - 4 - \left(x^2 - 4x\right)\right)dx$$

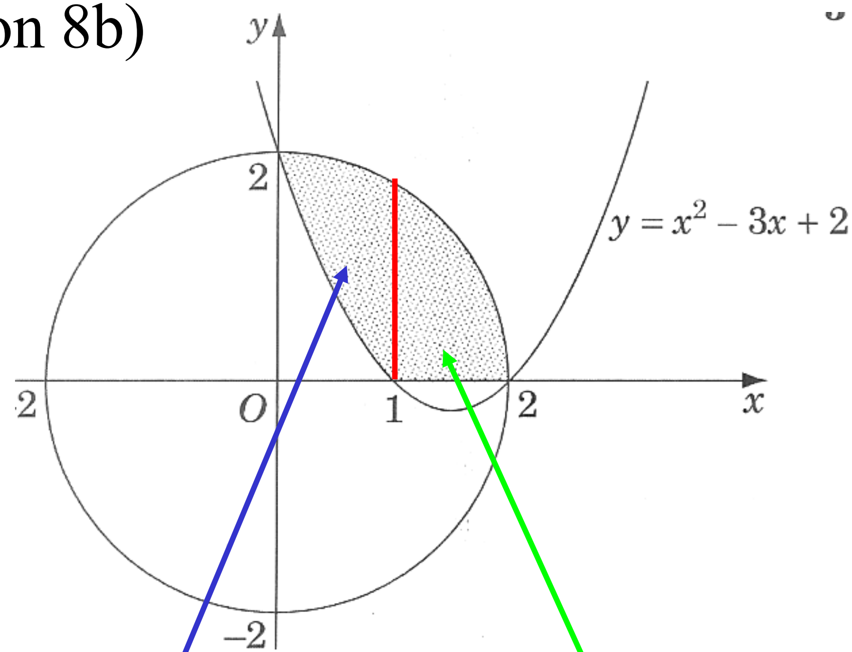
$$= \int_1^4 \left(-x^2 + 5x - 4\right)dx$$

$$= \left[-\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x\right]_1^4$$

$$= -\frac{1}{3}(4)^3 + \frac{5}{2}(4)^2 - 4(4) - \left\{-\frac{1}{3}(1)^3 + \frac{5}{2}(1)^2 - 4(1)\right\}$$

$$= \frac{9}{2} \text{ units}^2$$

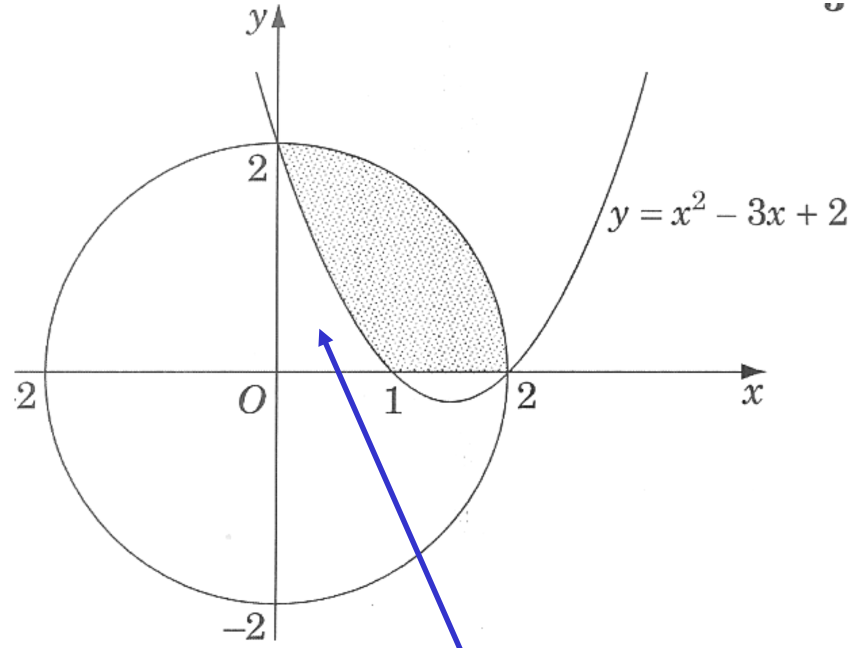
2005 HSC Question 8b) **(3)**



The shaded region in the diagram is bounded by the circle of radius 2 centred at the origin, the parabola $y = x^2 - 3x + 2$, and the x axis.
By considering the difference of two areas, find the area of the shaded region.

Note: area must be broken up into two areas, due to the different boundaries.

Area between circle and parabola and area between circle and x axis



It is easier to subtract the area under the parabola from the quadrant.

$$A = \frac{1}{4}\pi(2)^2 - \int_0^1 \left(x^2 - 3x + 2\right)dx$$

$$= \pi - \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x\right]_0^1$$

$$= \pi - \left\{\frac{1}{3}(1)^3 - \frac{3}{2}(1)^2 + 2(1) - 0\right\}$$

$$= \left(\pi - \frac{5}{6}\right)\text{units}^2$$

Exercise 5F; 1fgil, 2fh, 3bd, 4bd, 8be, 9d, 10a ii, b iii, 11, 12, 17

Exercise 5G; 1g, 2c, 3b, 4b, 8c, 11c, 12, 15, 16, 19