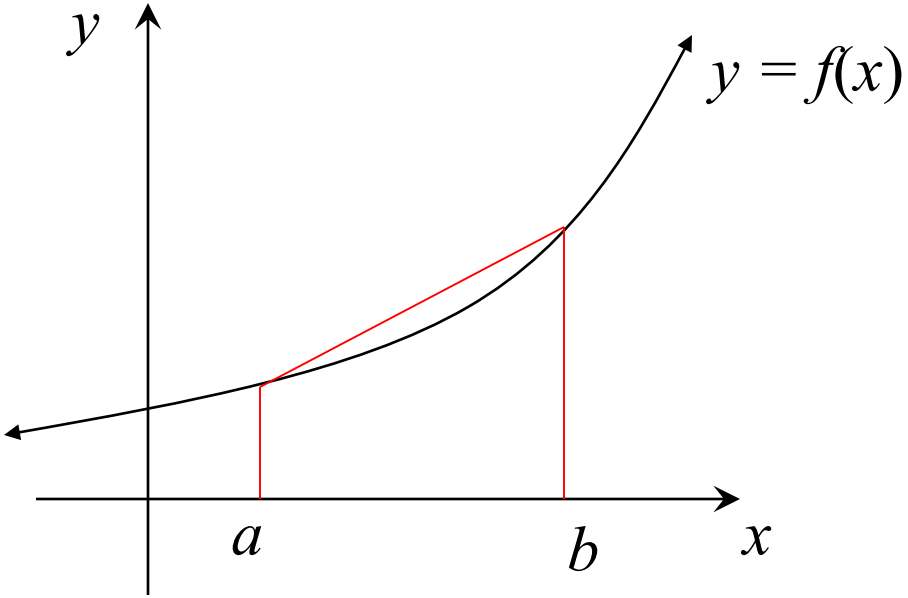
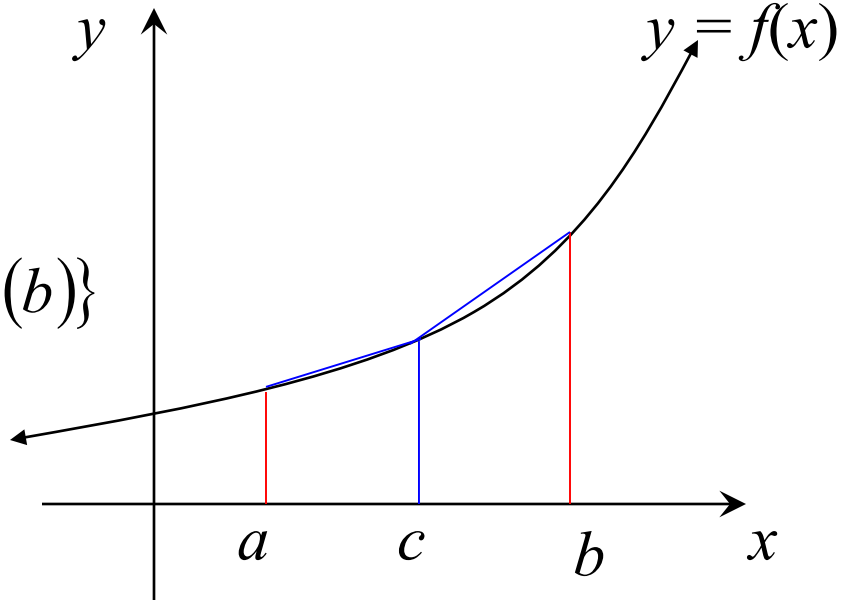
# *Trapezoidal Rule*

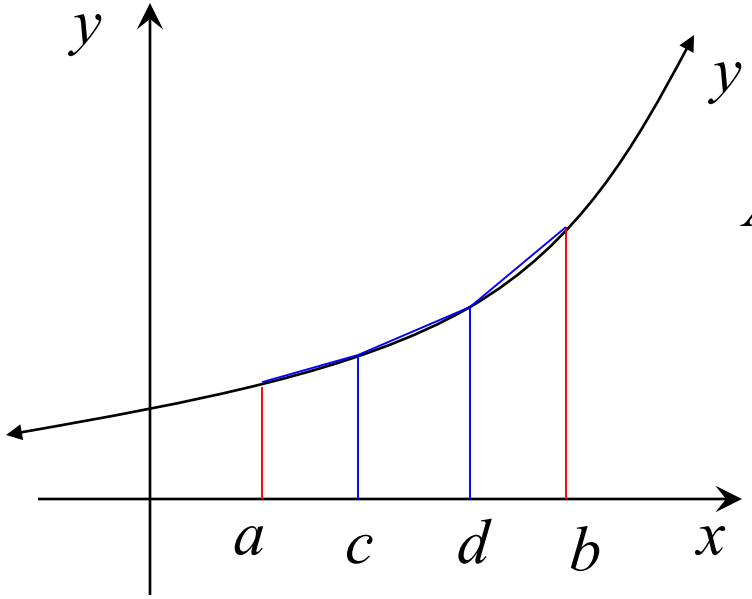



$$A \approx \frac{b-a}{2}\{f(a)+f(b)\}$$

$$A \approx \frac{c-a}{2}\{f(a)+f(c)\}+\frac{b-c}{2}\{f(c)+f(b)\}$$

$$= \frac{c-a}{2}\{f(a)+2f(c)+f(b)\}$$






$$A \approx \frac{c-a}{2}\{f(a)+f(c)\}+\frac{d-c}{2}\{f(c)+f(d)\}$$

$$+\frac{b-d}{2}\{f(d)+f(b)\}$$

$$=\frac{c-a}{2}\{f(a)+2f(c)+2f(d)+f(b)\}$$

In general;

$$\text{Area} = \int_a^b f(x)dx$$

$$\approx \frac{h}{2}\{y_0 + 2y_{others} + y_n\}$$

where $h = \dfrac{b-a}{n}$

$n$ = number of trapeziums

**NOTE:** there is always one more function value than interval

e.g. Use the Trapezoidal Rule with 4 intervals to estimate the area under the curve $y = \left(4 - x^2\right)^{\frac{1}{2}}$, between $x = 0$ and $x = 2$ (correct to 3 decimal points)

$$h = \frac{b-a}{n}$$

$$= \frac{2-0}{4}$$

$$= 0.5$$

| | 1 | 2 | 2 | 2 | 1 |
|---|---|---|---|---|---|
| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
| $y$ | 2 | 1.9365 | 1.7321 | 1.3229 | 0 |

$$\text{Area} \approx \frac{h}{2}\left\{y_0 + 2y_{others} + y_n\right\}$$

$$= \frac{0.5}{2}\left\{2 + 2\left(1.9365 + 1.7321 + 1.3229\right) + 0\right\}$$

$$= 2.996 \text{ units}^2 \qquad \left(exact\ value\ =\ \pi\right)$$

$$\% \text{ error} = \frac{3.142 - 2.996}{3.142} \times 100$$

$$= 4.6\%$$

# *Alternative working out!!!*

| | 1 | 2 | 2 | 2 | 1 |
|---|---|---|---|---|---|
| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
| $y$ | 2 | 1.9365 | 1.7321 | 1.3229 | 0 |

$$\text{Area} \approx \frac{2+2(1.9365+1.7321+1.3229)+0}{1+2+2+2+1} \times (2-0)$$

$$= 2.996 \text{ units}^2$$

e.g. **2020 HSC Question 20**

Kenzo is driving his car along a road while his friend records the velocity of the car, $v(t)$, in km/h every minute over a 5-minute period. The table gives the velocity $v(t)$ at time $t$ hours

| $t$ | 0 | $\frac{1}{60}$ | $\frac{2}{60}$ | $\frac{3}{60}$ | $\frac{4}{60}$ | $\frac{5}{60}$ |
|---|---|---|---|---|---|---|
| $v(t)$ | 60 | 55 | 65 | 68 | 70 | 67 |

The distance covered by the car over the 5-minute period is given by

$$\int_0^{\frac{5}{60}} v(t)dt$$

Use the trapezoidal rule and the velocity at each of the six time values to find the approximate distance in kilometres the car has travelled in the 5-minute period. Give your answer correct to one decimal place.

| | 1 | 2 | 2 | 2 | 2 | 1 |
|---|---|---|---|---|---|---|
| $t$ | 0 | $\frac{1}{60}$ | $\frac{2}{60}$ | $\frac{3}{60}$ | $\frac{4}{60}$ | $\frac{5}{60}$ |
| $v(t)$ | 60 | 55 | 65 | 68 | 70 | 67 |

$$\text{Distance} \approx \frac{h}{2}\{y_0 + 2y_{\text{others}} + y_n\}$$

$$= \frac{\frac{1}{60}}{2}\{60 + 2(55 + 65 + 68 + 70) + 67\}$$

$$= \frac{643}{120}$$

$$= 5.4 \text{ km} \quad \text{(to 1 dp)}$$

***OR***

$$\text{Distance} \approx \frac{60 + 2(55 + 65 + 68 + 70) + 87}{1 + 2 + 2 + 2 + 2 + 1} \times \left(\frac{5}{60} - 0\right)$$

$$= 5.3583333\ldots$$

$$= 5.4 \text{ km (to 1 dp)}$$

**Exercise 5H; 4, 5, 6, 7, 8ab, 10, 12, 14, 15**