Maxima & Minima Problems

When solving word problems we need to;

(1) Reduce the problem to a set of equations
(there will usually be two)

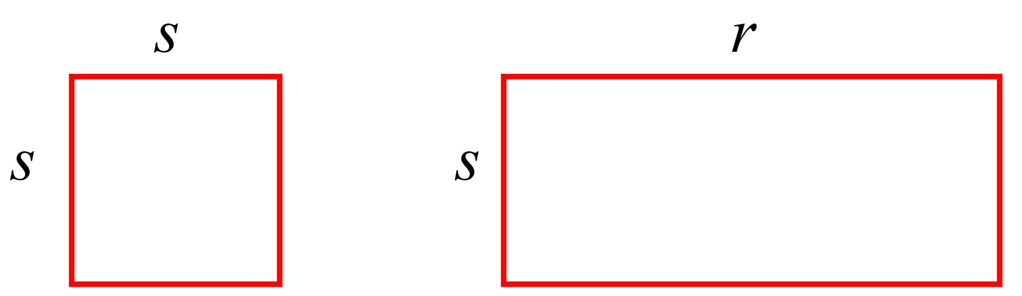
a) one will be what you are maximising/minimising

b) one will be some information given in the problem

(2) Rewrite the equation we are trying to minimise/maximise with only one variable

(3) Use calculus to solve the problem

e.g. A rope 36 metres in length is cut into two pieces. The first is bent to form a square, the other forms a rectangle with one side the same length as the square.
Find the length of the square if the sum of the areas is to be a maximum.



$$A = s^2 + rs \ldots (1) \qquad 6s + 2r = 36 \ldots (2)$$

make r the subject in (2)

$$2r = 36 - 6s$$

$$r = 18 - 3s$$

substitute into (1)

$$A = s^2 + (18 - 3s)s$$

$$A = s^2 + 18s - 3s^2$$

$$A = 18s - 2s^2$$

$$A = 18s - 2s^2$$

$$\frac{dA}{ds} = 18 - 4s$$

$$\frac{d^2A}{ds^2} = -4$$

Stationary points occur when $\frac{dA}{ds} = 0$

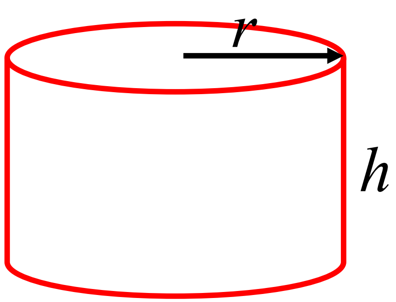
$$\text{i.e. } 18 - 4s = 0$$

$$s = \frac{9}{2}$$

when $s = \frac{9}{2}, \frac{d^2A}{ds^2} = -4 < 0$

$\therefore$ when the side length of the square is $4\frac{1}{2}$ metres, the area is a maximum

(ii) An open cylindrical bucket is to made so as to hold the largest possible volume of liquid.
If the total surface area of the bucket is 12π units^2, find the height, radius and volume.



$$V = \pi r^2 h \ldots (1)$$

$$12\pi = \pi r^2 + 2\pi r h \ldots (2)$$

make h the subject in (2)

$$2\pi r h = 12\pi - \pi r^2$$

$$h = \frac{12 - r^2}{2r}$$

substitute into (1)

$$V = \pi r^2 \left(\frac{12 - r^2}{2r} \right)$$

$$V = 6\pi r - \frac{1}{2}\pi r^3$$

$$V = 6\pi r - \frac{1}{2}\pi r^3$$

$$\frac{dV}{dr} = 6\pi - \frac{3}{2}\pi r^2$$

$$\frac{d^2V}{dr^2} = -3\pi r$$

Stationary points occur when $\frac{dV}{dr} = 0$

$$\text{i.e. } 6\pi - \frac{3}{2}\pi r^2 = 0$$

$$\frac{3}{2}\pi r^2 = 6\pi$$

$$r^2 = 4$$

$$r = \pm 2$$

when $r = 2, \frac{d^2V}{dr^2} = -6\pi < 0$

$\therefore$ when $r = 2$, V is a maximum

when $r = 2, h = \frac{12 - 2^2}{2(2)}$

$$= 2$$

$$V = \pi(2)^2(2)$$

$$= 8\pi$$

$\therefore$ maximum volume is 8π units^3 when the radius and height are both 2 units

Exercise 4H; 1, 5, 6, 7, 9, 11, 12, 17, 19, 21

Exercise 4I; 2, 5, 7, 9, 11, 13, 14