Binomial Products

Bi = 2 terms *nomial* = terms

a single term in a binomial product is called a **binomium**

e.g. $(x+6)(x+1) = x^2 + x + 6x + 6$

$= x^2 + 7x + 6$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

In general :

$$(a+b+c+\ldots)^2 = a^2 + b^2 + c^2 + \ldots + 2ab + 2ac + 2bc + \ldots$$

$$(a+b)(a-b) = a^2 - b^2$$

$(a+b)^2 \qquad a^2 \quad +2ab \quad +b^2$

e.g. $(i)\ (x+2)^2 = x^2 + 2(x)(2) + 2^2$

$= x^2 + 4x + 4$

$(ii)\ (3x-4)^2 = 9x^2 - 24x + 16$

$(iii)\ (2p-5)(2p+5) = 4p^2 - 25$

2 terms × 3 terms $\qquad \therefore$ answer has 6 terms

$(iv)\ (a+2)(a^2-3a+7) = a^3 - 3a^2 + 7a + 2a^2 - 6a + 14$

$= a^3 - a^2 + a + 14$

Method of Detached Coefficients

e.g. $(i)\ \left(2x^3-4x^2-5\right)\left(3x^2+4x-2\right)=6x^5-4x^4-20x^3-7x^2-20x+10$

1. List **all** the coefficients of the first factor, starting with highest power
2. List **all** the coefficients of the second factor, starting with highest power
3. Multiply all of the first row by the **first** number in the second row, writing the first result under the **first** number
4. Multiply all of the first row by the **second** number in the second row, writing the first result under the **second** number, and so on….
5. **Add** down the columns
6. The result is the coefficients of the answers, starting with the highest power

highest power
equals
highest power
times
highest power

2	−4	0	−5		
3	4	−2			
6	−12	0	−15		
	8	−16	0	−20	
		−4	8	0	10
6	−4	−20	−7	−20	10

(ii) $(a+2)(a^2-3a+7) = a^3-a^2+a+14$

$$\begin{array}{cccc} 1 & -3 & 7 & \\ 1 & 2 & & \\ \hline 1 & -3 & 7 & \\ & 2 & -6 & 14 \\ \hline 1 & -1 & 1 & 14 \end{array}$$

each term in this quartic is the product of four pronumerals it is "missing" the x^2y^2 term

it is multiplied by a quadratic where each term is the product of two pronumerals

(iii) $(3x^4+2x^3y+4xy^3+2y^4)(2x^2-y^2)$

$= 6x^6+4x^5y-3x^4y^2+6x^3y^3+4x^2y^4-4xy^5-2y^6$

the answer is a sextic where each term is the product of six pronumerals

$$\begin{array}{ccccccc} 3 & 2 & 0 & 4 & 2 & & \\ 2 & 0 & -1 & & & & \\ \hline 6 & 4 & 0 & 8 & 4 & & \\ & -3 & -2 & 0 & -4 & -2 & \\ \hline 6 & 4 & -3 & 6 & 4 & -4 & -2 \end{array}$$

Exercise 1B; 3agk, 4fl, 6mqv, 7ac, 8, 9abd, 10bd, 11b, 12*, 13b*, 14*