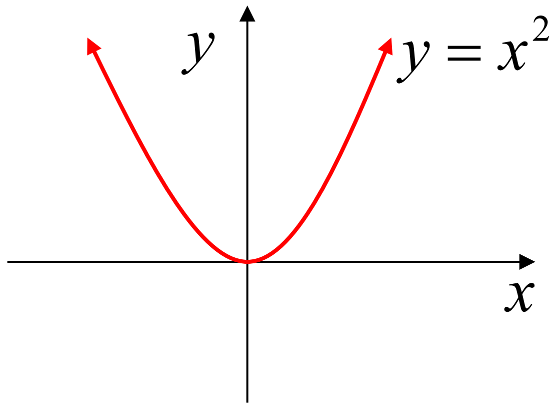
Quadratic Function



The linear function and the **quadratic function** are the building blocks of all polynomials

Every polynomial can be factorised down to a combination of linear and quadratic factors.

All quadratics can be transformed from the basic equation $y = x^2$ using transformations (translations, rotations, reflections) and/or dilations.

Recognising the quadratic function

$$y = ax^2 + bx + c$$

power '1'   power '2'

- terms contain at most one variable, one variable is to the power of one, the other variable hs a term to the power of two

Quadratics and Completing the Square

a measures concavity

$$\boxed{y = a(x-h)^2 + k}$$

vertex is (h, k)

e.g. Sketch the parabola $y = x^2 + 8x + 12$

$$y = x^2 + 8x + 12$$
$$= (x+4)^2 - 4$$

$\therefore$ vertex is $(-4, -4)$

x intercepts

$$(x+4)^2 - 4 = 0$$
$$(x+4)^2 = 4$$
$$x + 4 = \pm 2$$
$$x = -4 \pm 2$$
$$x = -6 \text{ or } x = -2$$

$\therefore$ x intercepts are $(-6, 0)$ and $(-2, 0)$

(ii) Write down the quadratic with roots 2 and 8 and vertex $(5,3)$

$$y = k\left\{(x-5)^2\right\} + 3$$

$$(2,0):\ 0 = k\left\{(2-5)^2\right\} + 3$$

$$9k = -3$$

$$k = -\frac{1}{3}$$

$$y = -\frac{1}{3}\left\{(x-5)^2\right\} + 3$$

$$y = -\frac{1}{3}\left(x^2 - 10x + 16\right)$$

Quadratics and the Discriminant

$$\boxed{\Delta = b^2 - 4ac}$$

$$\boxed{\begin{aligned} \text{vertex} &= \left(\frac{-b}{2a}, \frac{-\Delta}{4a}\right) \\ \text{zeroes} &= \frac{-b \pm \sqrt{\Delta}}{2a} \end{aligned}}$$

Note: if $\Delta < 0,$ no x intercepts

$\Delta = 0,$ one x intercept

$\Delta > 0,$ two x intercepts

e.g. Sketch the parabola $y = x^2 + 8x + 12$

$$\Delta = 8^2 - 4(1)(12)$$

$$= 16$$

$$\therefore \text{ vertex} = \left(-\frac{8}{2}, -\frac{16}{4}\right)$$

$$= \underline{(-4, -4)}$$

The Discriminant

The discriminant tells us whether the roots are rational or irrational

$\Delta > 0$: two different real roots (cuts the x axis twice)

$\Delta = 0$: two equal real roots (touches the x axis once)

$\Delta < 0$: no real roots (never touches the x axis)

Δ is a perfect square : roots are rational

e.g. (i) describe the roots of

$$2x^2 + 6x - 3 = 0$$

$$\Delta = 6^2 - 4(2)(-3)$$

$$= 60 > 0$$

$\therefore$ two different, real, irrational roots

(ii) Find the values of k if

$x^2 - 4x + 2k = 0$ has no real roots

unreal roots occur when $\Delta < 0$

$$i.e.\ (-4)^2 - 4(2k) < 0$$

$$16 - 8k < 0$$

$$k > 2$$

Exercise 3E;1a, 2c, 3ace, 4b, 5ac,6bc, 7c, 8, 9be, 10a, 11ace, 12be, 13ac, 14bdf, 15bc, 16, 19

Exercise 3F; 1a, 2adf, 5a, 6, 7, 8,9ace, 10a, 11, 12b, 14