

# *Differentiating Exponentials*

$$y = e^{f(x)}$$
$$\frac{dy}{dx} = f'(x)e^{f(x)}$$

$$y = a^{f(x)}$$
$$\frac{dy}{dx} = f'(x)(\log a)a^{f(x)}$$

e.g. (i)  $y = e^x$

$$\frac{dy}{dx} = e^x$$

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(ii)  $y = e^{5x}$

$$\frac{dy}{dx} = 5e^{5x}$$

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(iii)  $y = e^{4x+3}$

$$\frac{dy}{dx} = 4e^{4x+3}$$

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(iv)  $y = e^{x^2+3x+2}$

$$\frac{dy}{dx} = (2x+3)e^{x^2+3x+2}$$

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$$\begin{aligned}
 \text{(v)} \quad y &= 3x^2 e^{4x} \\
 \frac{dy}{dx} &= (3x^2)(4e^{4x}) + (e^{4x})(6x) \\
 &= 12x^2 e^{4x} + 6xe^{4x} \\
 &= \underline{6xe^{4x}(2x+1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad y &= \frac{e^x}{e^x + 3} \\
 \frac{dy}{dx} &= \frac{(e^x + 3)(e^x) - (e^x)(e^x)}{(e^x + 3)^2} \\
 &= \frac{e^{2x} + 3e^x - e^{2x}}{(e^x + 3)^2} \\
 &= \underline{\frac{3e^x}{(e^x + 3)^2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad y &= (e^{3x} + 2)^7 \\
 \frac{dy}{dx} &= 7(e^{3x} + 2)^6 (3e^{3x}) \\
 &= \underline{21e^{3x}(e^{3x} + 2)^6}
 \end{aligned}$$

(viii) Find the tangent to  $y = e^{2x} + 1$  at the point  $(1, e^2 + 1)$

$$y = e^{2x} + 1$$

$$\frac{dy}{dx} = 2e^{2x}$$

$$\text{when } x = 1, \frac{dy}{dx} = 2e^2$$

$$y - (e^2 + 1) = 2e^2(x - 1)$$

$$y - e^2 - 1 = 2e^2x - 2e^2$$

$$\underline{2e^2x - y - e^2 + 1 = 0}$$

$$(ix) \quad y = 4^{x^2}$$

$$\frac{dy}{dx} = 2x(\log 4)4^{x^2}$$

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$$(x) \quad y = \log x$$

$$x = e^y$$

$$\frac{dx}{dy} = e^y$$

$$dy$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$= \frac{1}{x}$$

$$= \frac{1}{x}$$

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## e.g. 2025 HSC Question 16

Consider the function  $f(x) = \frac{x^2}{e^x}$

a) Find the stationary points of the function and determine their nature

numerator is left unfactored as it will make finding the second derivative easier

$$\begin{aligned} f'(x) &= \frac{(e^x)(2x) - (x^2)(e^x)}{e^{2x}} & f'(x) &= \frac{(e^x)(2 - 2x) - (2x - x^2)(e^x)}{e^{2x}} \\ &= \frac{2x - x^2}{e^x} & &= \frac{2 - 4x + x^2}{e^x} \end{aligned}$$

stationary points occur when  $f'(x) = 0$

$$\text{i.e. } x(2 - x) = 0$$

$$x = 0 \quad \text{or} \quad x = 2$$

$$\text{when } x = 0, f'(0) = \frac{2}{1} > 0$$

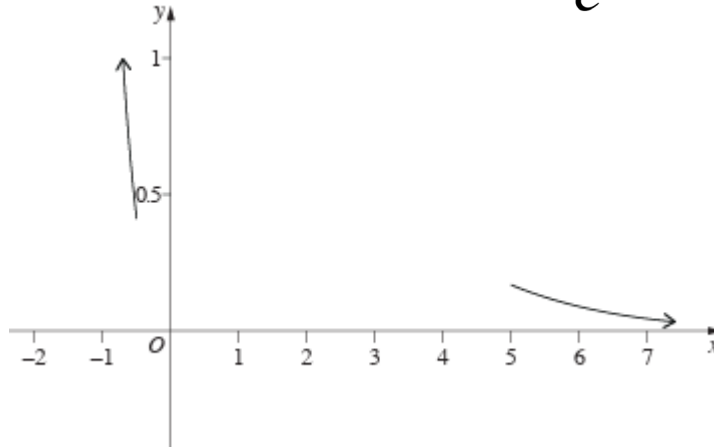
$$\text{when } x = 2, f'(2) = \frac{2 - 8 + 4}{1}$$

$$= -\frac{2}{e^2} < 0$$

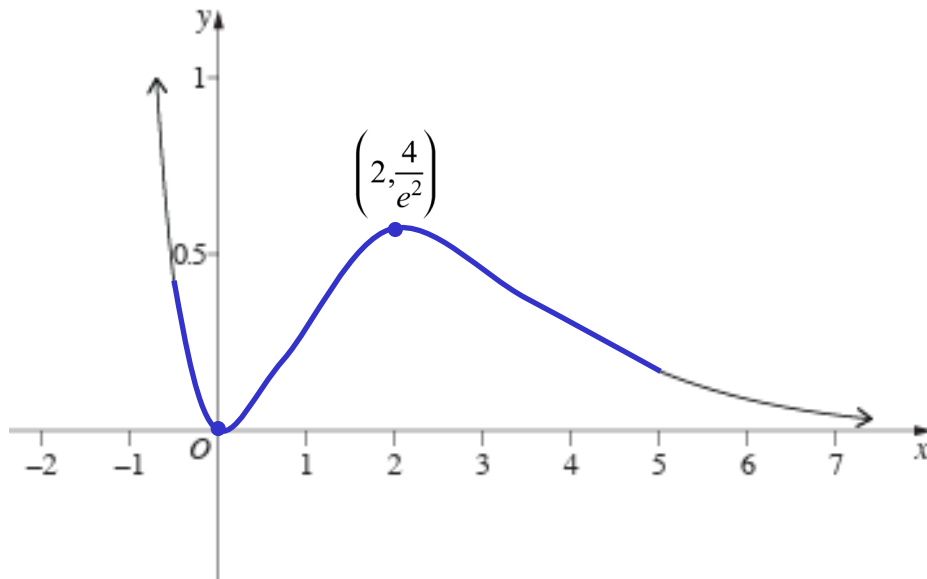
$\therefore (0, 0)$  is a minimum turning point

$\therefore \left(2, \frac{4}{e^2}\right)$  is a maximum turning point

b) A partially completed graph of  $f(x) = \frac{x^2}{e^x}$  is shown.



Use your answer from part a) to complete the graph



**Exercise 6B; 1adg, 2df,  
3bdfh, 4b, 5acg, 6bg, 7ah,  
8aeg, 9d, 10ac, 11b, 12e, 14bf,  
15e, 16bf, 17acegj, 20**

**Exercise 6C; 3, 8, 9, 13,  
14, 16a, 17, 18**

**Exercise 6K; 4a, 5c**