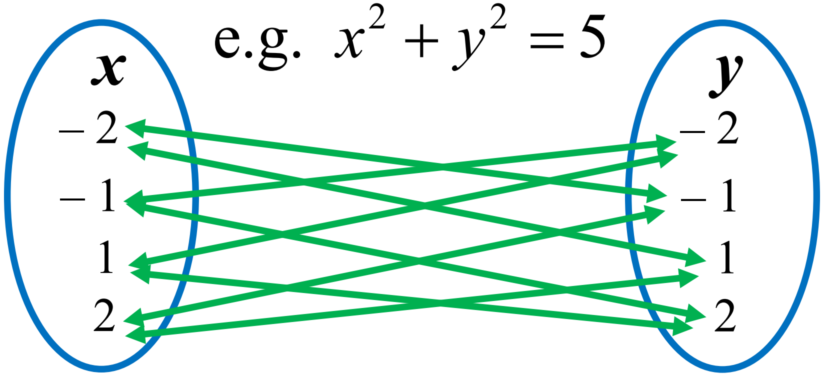
# *Functions*

## Definitions:

**relation** – a rule that maps between two sets of values

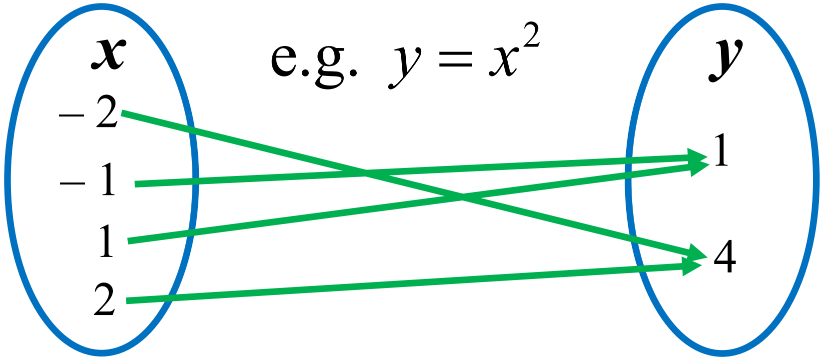


e.g. $x^2 + y^2 = 5$

a relation can be represented by

- an algebraic formula
- a table of values
- a set of ordered pairs
- a graph

**function** – a type of relation that uniquely maps from one set of values to another set of values



e.g. $y = x^2$

**independent variable** – the "***input***" of the function

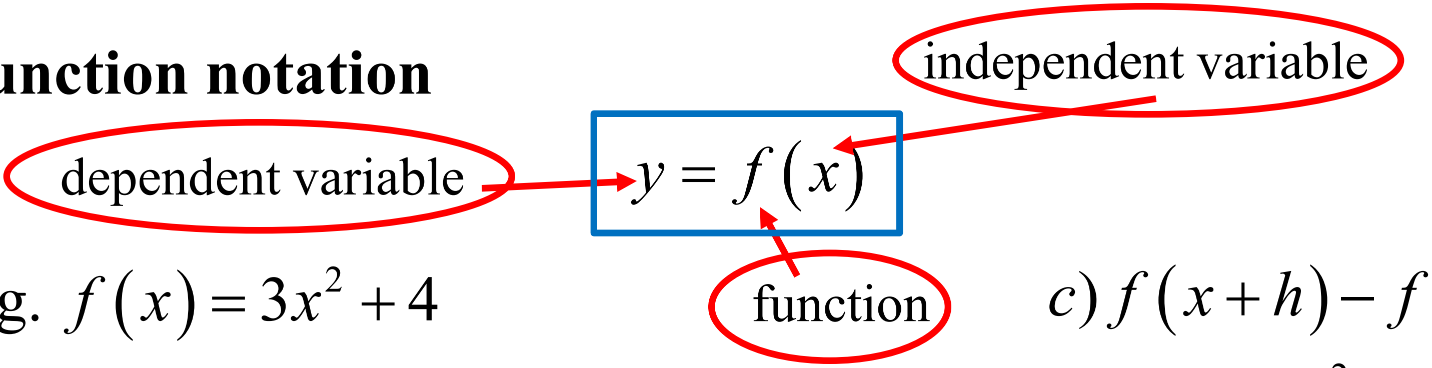
**domain**– *natural domain*: all possible values of the independent variable that can be substituted into the function

*restricted domain:* domains are restricted when some values are not required e.g. if independent variable represents time, then only $t \geq 0$ would be needed

**dependent variable**– the "***output***" of the function, its value *depends* upon the value of the independent variable

**range**– all possible values the dependent variable can take; obtained by substituting every value in the domain

## function notation



e.g. $f(x) = 3x^2 + 4$

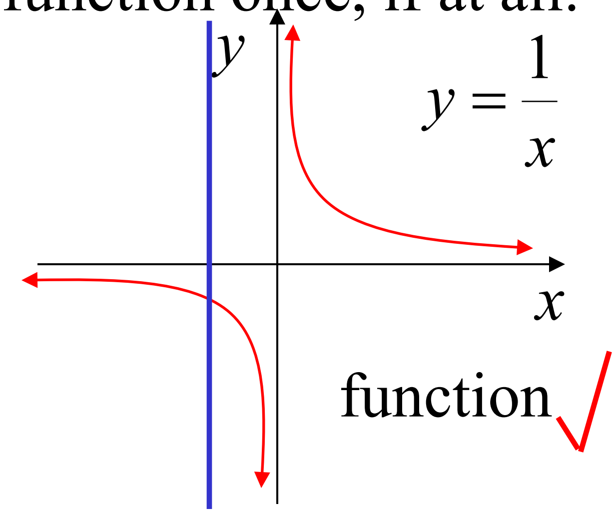
$$a)\ f(5) = 3(5)^2 + 4$$
$$= 75 + 4$$
$$\underline{= 79}$$

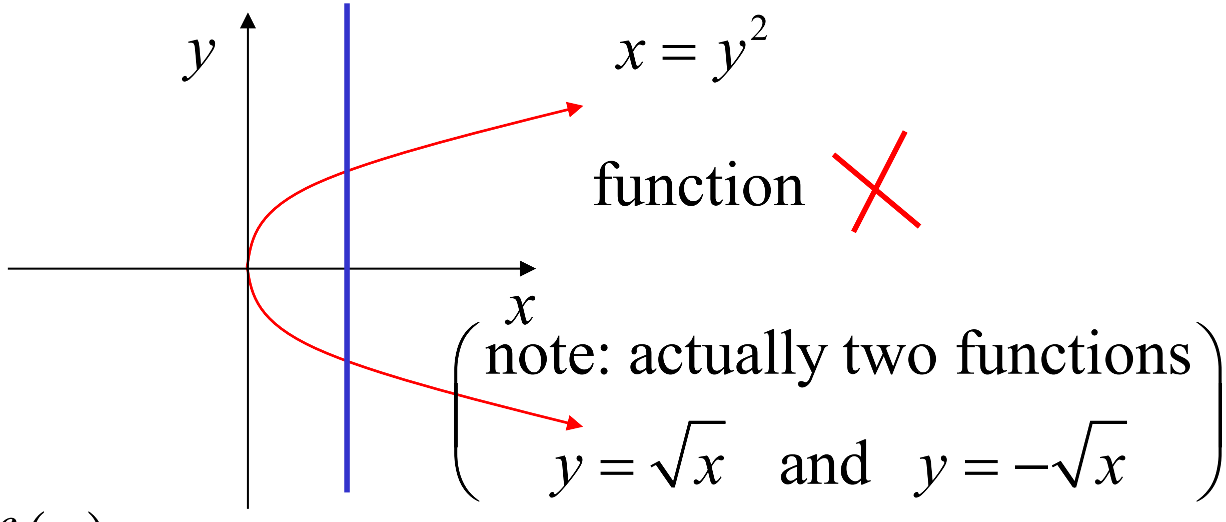
$$b)\ f(a) = \underline{3a^2 + 4}$$

$$c)\ f(x+h) - f(x)$$
$$= 3(x+h)^2 + 4 - (3x^2 + 4)$$
$$= 3x^2 + 6xh + 3h^2 + 4 - 3x^2 - 4$$
$$\underline{= 6xh + 3h^2}$$

## Vertical Line Test for Functions

If a straight line is drawn parallel to the $y$ axis, it will only cross a function once, if at all.



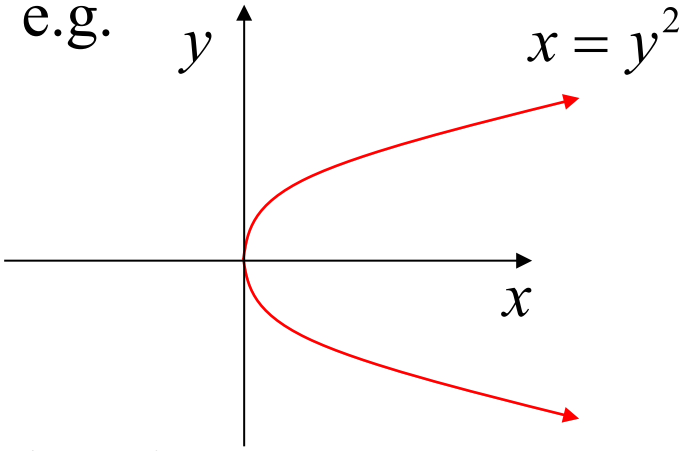


$\left( \text{note: actually two functions} \quad y = \sqrt{x} \quad \text{and} \quad y = -\sqrt{x} \right)$

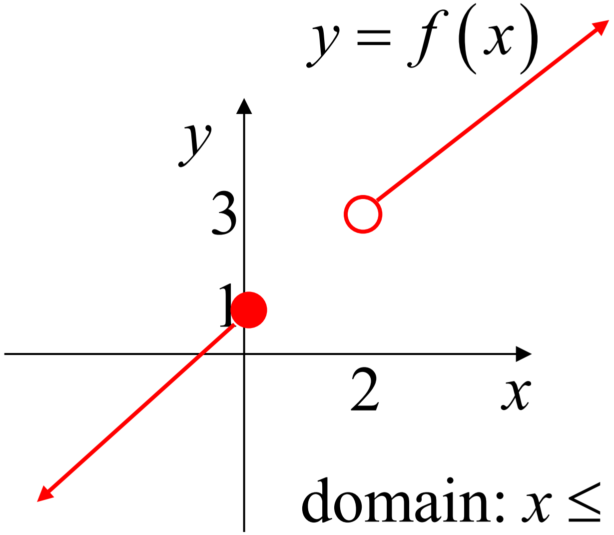
## Domain and Range $y = f(x)$

Finding Domain: Geometrically it can be read from the graph

e.g.



domain: $x \geq 0$



domain: $x \leq 0$ and $x > 2$

Algebraically; things to look for:

1. Fractions: bottom of fraction $\neq 0$

e.g. $(i)\ y = \dfrac{1}{x}$

$$x \neq 0$$

domain: all real $x$ except $x = 0$

$(ii)\ y = \dfrac{1}{x^2 - 1}$

$$x^2 - 1 \neq 0$$

$$x^2 \neq 1$$

$$x \neq \pm 1$$

domain: all real $x$ except $x = \pm 1$

$(iii)\ y = \dfrac{4x}{x-1} + \dfrac{3}{7-x}$

$$x - 1 \neq 0 \qquad 7 - x \neq 0$$

$$x \neq 1 \qquad x \neq 7$$

domain: all real $x$ except $x = 1$ or $7$

**2. Root Signs:** you can't find the square root of a negative number.

e.g. $(i)\ y=\sqrt{4-x^2}$

$$4-x^2\geq 0$$

$$x^2\leq 4$$

domain: $-2\leq x\leq 2$

$(ii)\ y=\sqrt{x+3}-\sqrt{5-x}$

$x+3\geq 0$ $\quad$ $5-x\geq 0$

$x\geq -3$ $\quad$ $x\leq 5$

domain: $-3\leq x\leq 5$

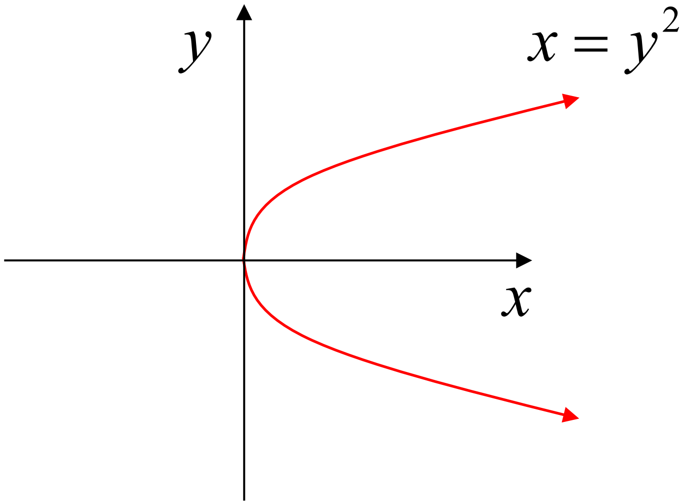
$(iii)\ y=\dfrac{1}{\sqrt{x+2}}$

$$x+2>0$$

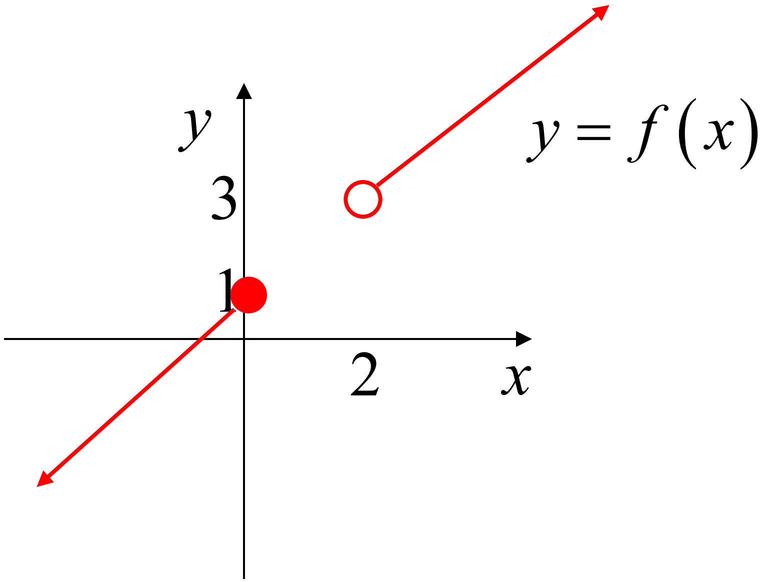
domain: $x>-2$

# Range: Geometrically

e.g.



range: all real $y$



range: $y \leq 1$ and $y > 3$

Algebraically; things to look for:

1. Maximum/Minimum values: even powers and absolute values are always $\geq$ 0

e.g. $(i)\ y = x^2$

range: $y \geq 0$

$(ii)\ y = x^2 + 3$

$y \geq 0 + 3$

range: $y \geq 3$

$(iii)\ y = 5 - x^2$

$y \leq 5 - 0$

range: $y \leq 5$

$(iv)\ y = |x + 2|$

range: $y \geq 0$

$(v)\ y = |x + 2| - 5$

$y \geq 0 - 5$

range: $y \geq -5$

**2. Restrictions on Domain:** sub in endpoints and centre of domain

e.g. $y=\sqrt{4-x^2}$

domain: $-2\leq x\leq 2$

when $x=2,\ y=\sqrt{4-2^2}$
$=0$

when $x=0,\ y=\sqrt{4-0^2}$
$=2$

range: $0\leq y\leq 2$

**3. Fractions:** If you have a constant on the top of the fraction, fraction $\neq 0$

e.g.(i) $y=\dfrac{1}{x}$

$y\neq 0$

range: all real $y$ except $y=0$

(ii) $y=5+\dfrac{1}{x}$

$y\neq 5+0$

range: all real $y$ except $y=5$

(iii) $y=\dfrac{x+7}{x+4}$

$$\begin{array}{r} 1\phantom{)} \\ x+4\overline{)x+7} \\ \underline{x+4} \\ 3 \end{array}$$

$y=1+\dfrac{3}{x+4}$

$y\neq 1+0$

range: all real $y$ except $y=1$

**Exercise 3A; 1b, 2c, 3d, 4d, 7ac, 8a, 9b, 10bd, 11, 12bc, 13bd, 14ac, 15ac, 16, 17, 18**

**Exercise 3B; 2, 3, 4b, 5b, 6d, 7, 9be, 10, 11, 12, 14adf, 15, 16**