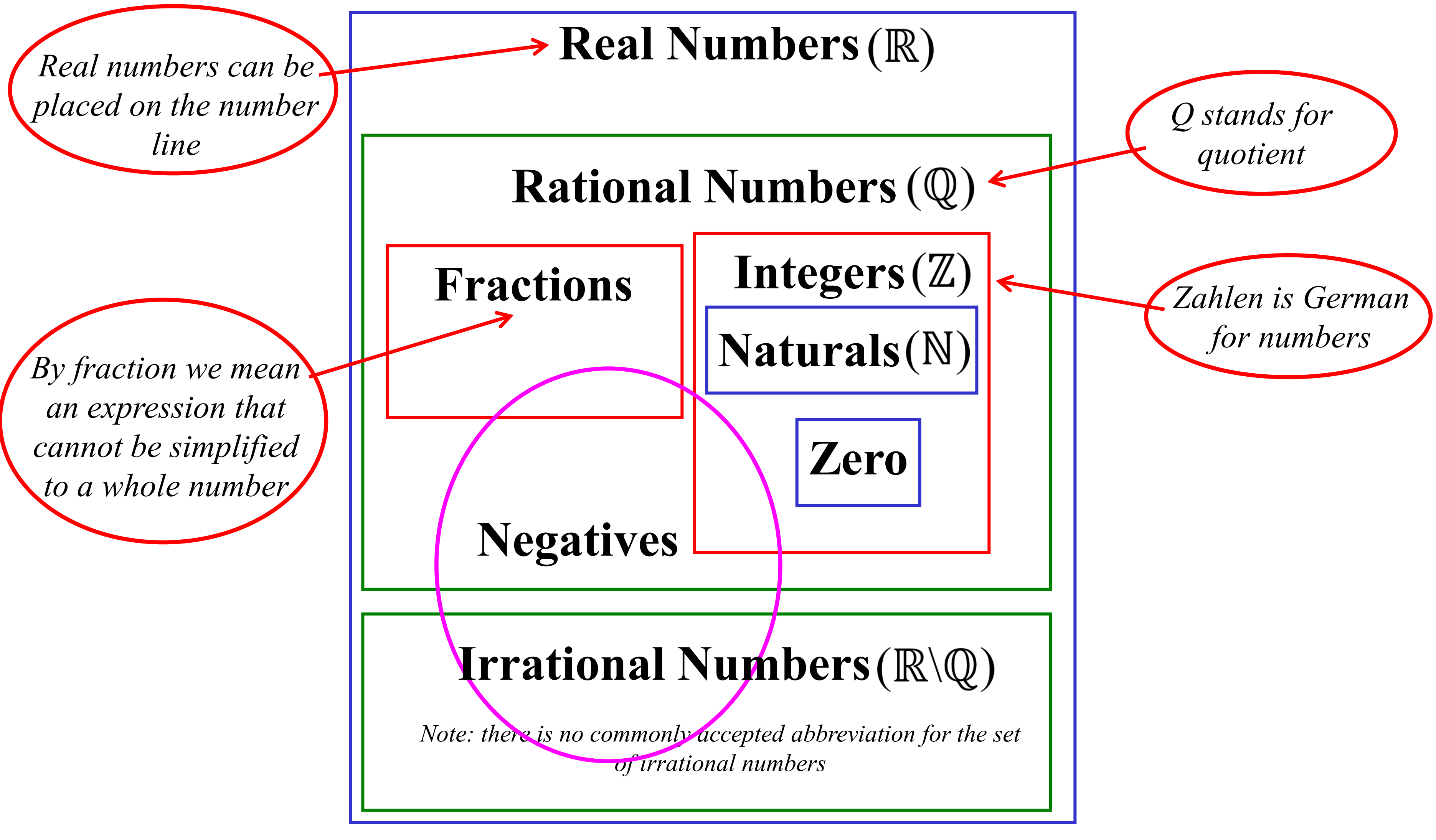
Real Numbers



1. Prime Factors

Every natural number can be written as a product of its prime factors.

e.g. $324 = 4 \times 81$

$= 2^2 \times 3^4$

2. Highest Common Factor (HCF)

1) Write both numbers in terms of its prime factors

2) Take out the common factors

e.g. 1176 and 252

$1176 = 6 \times 196$

$= 3 \times 2 \times 49 \times 4$

$= 3 \times 2^3 \times 7^2$

$252 = 4 \times 63$

$= 4 \times 9 \times 7$

$= 2^2 \times 3^2 \times 7$

$HCF = 2^2 \times 3 \times 7$

$= 84$

When factorising, remove the lowest power

3. Lowest Common Multiple (LCM)

1) Write both numbers in terms of its prime factors

2) Write down all factors without repeating

e.g. 48 and 15

$$48 = 16 \times 3 = 2^4 \times 3$$

$$15 = 3 \times 5$$

$$LCM = 2^4 \times 3 \times 5 = 240$$

When creating a LCM, use the highest power

4. Divisibility Tests

2: even number

3: digits add to a multiple of 3

4: last two digits are divisible by 4

5: ends in a 5 or 0

6: divisible by 2 and 3

7: double the last digit and subtract from the other digits, answer is divisible by 7

8: last three digits are divisible by 8

9: sum of the digits is divisible by 9

10: ends in a 0

11: sum of even positioned digits = sum of odd positioned digits, or differ by a multiple of 11.

Representing Real Numbers

All real numbers can be placed on the number line and described;

- geometrically *(using a picture of the number line)*
- algebraically *(using an inequation or equation)*
- using interval notation *(often used when describing domain & range)*
- using set notation *(formal way of describing all possible numbers)*

Types of Intervals

(i) bounded: interval has two endpoints

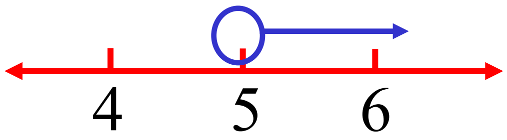
(ii) unbounded: interval has one endpoint

(iii) closed: all endpoints are included

(iv) open: an endpoint is not included

(v) degenerate: a single point

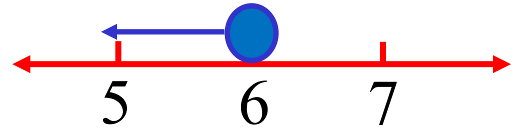
e.g. $(i)\ x > 5$



open unbounded interval

$(5,\infty) = \{x : x > 5\}$

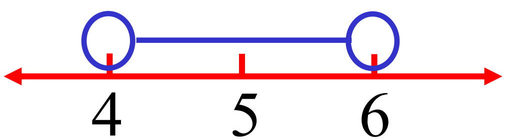
$(ii)\ x \le 6$



closed unbounded interval

$(-\infty,6] = \{x : x \le 6\}$

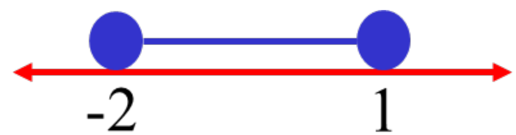
$(iii)\ 4 < x < 6$



open bounded interval

$(4,6) = \{x : 4 < x < 6\}$

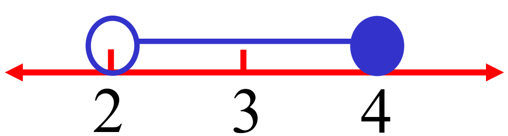
$(iv)\ -2 \le x \le 1$



closed bounded interval

$[-2,1] = \{x : -2 \le x \le 1\}$

$(v)\ 2 < x \le 4$



bounded interval

$(2,4] = \{x : 2 < x \le 4\}$

Rational Numbers

Rational numbers can be expressed in the form $\frac{a}{b}$ where a and b are integers.

Irrational Numbers

Irrational numbers are numbers which are not rational.

All irrational numbers can be expressed as a unique infinite decimal.

e.g. Prove $\sqrt{2}$ is irrational

"Proof by contradiction"

Assume $\sqrt{2}$ is rational

$\therefore \sqrt{2} = \dfrac{a}{b}$ where a and b are integers with no common factors

another way of saying this is "where a and b are **coprime**"

coprime numbers are numbers that have no common factor, they do not have to be prime
e.g. 14 and 15 are coprime
so are 2, 4 and 25

$b\sqrt{2} = a$

$2b^2 = a^2$

Thus a^2 must be divisible by 2

As prime factors of squares must appear in pairs, any square that is divisible by 2 is also divisible by 4

Thus a^2 must be divisible by 4

$\therefore\ 2b^2 = 4k$ where k is an integer

$b^2 = 2k$

So a^2 and b^2 are both divisible by 2 and must have a common factor

However, any prime factor of a square is also a factor of its square root

Yet a and b have no common factors

$\therefore \sqrt{2}$ is irrational, by contradiction

Exercise 2A; 1cdfikl, 2ad, 3bc, 4c, 5c, 6ace, 7bdf, 9, 10b, 12, 14, 15