

The Discriminant

$$\Delta = b^2 - 4ac$$

The discriminant tells us whether the roots are rational or irrational

$\Delta > 0$: two different real roots (cuts the x axis twice)

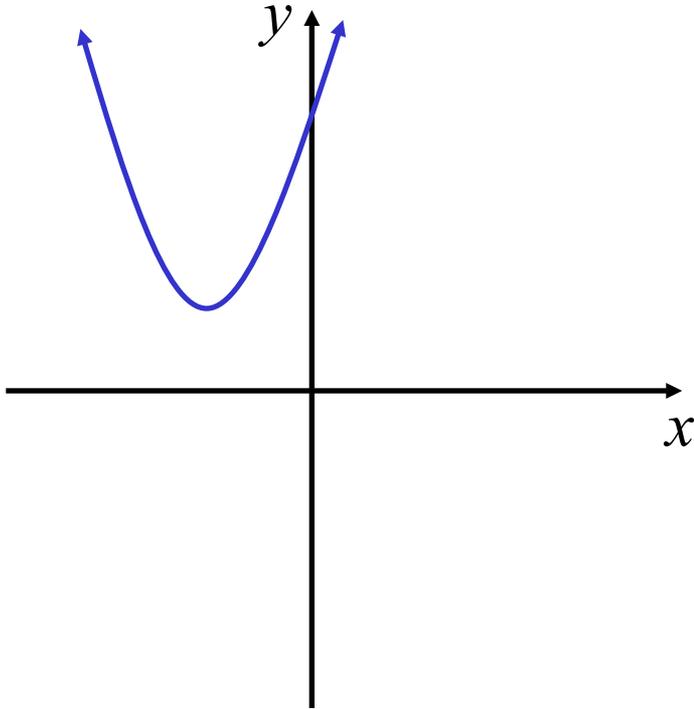
$\Delta = 0$: two equal real roots (touches the x axis once)

$\Delta < 0$: no real roots (never touches the x axis)

Δ is a perfect square : roots are rational

The Sign Of A Quadratic

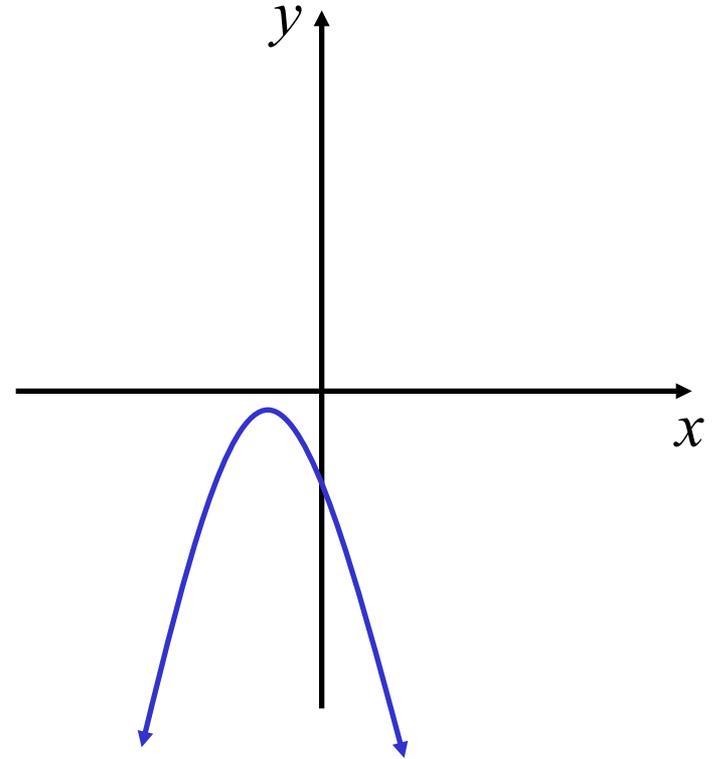
Positive Definite



$$a > 0 \quad , \quad \Delta < 0$$

this means that the quadratic
function is definitely
positive

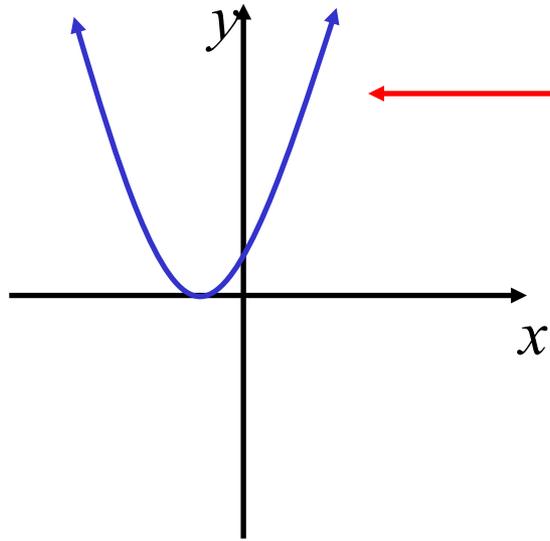
Negative Definite



$$a < 0 \quad , \quad \Delta < 0$$

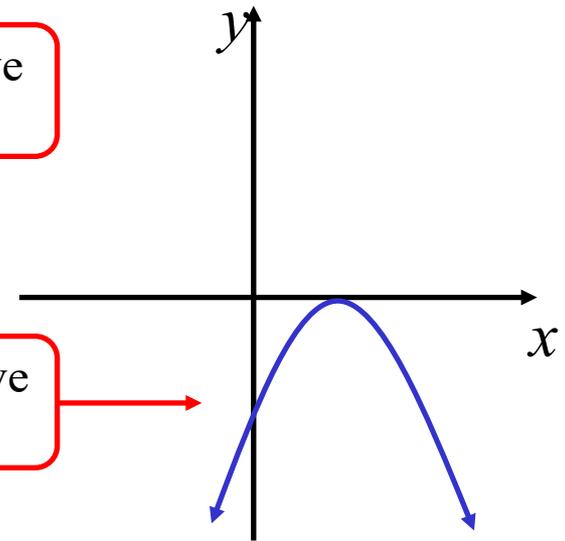
this means that the quadratic
function is definitely
negative

Indefinite



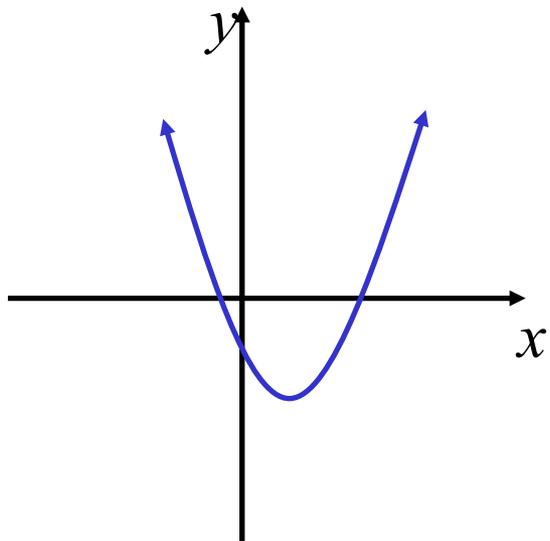
$$a > 0, \Delta = 0$$

Also known as positive semi-definite

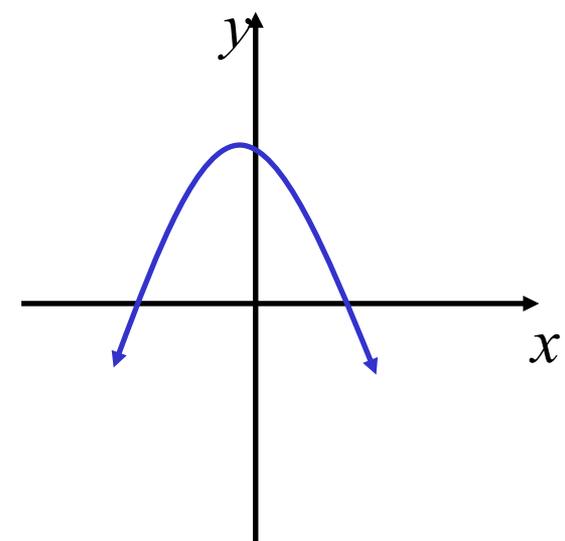


$$a < 0, \Delta = 0$$

Also known as negative semi-definite



$$a > 0, \Delta > 0$$



$$a < 0, \Delta > 0$$

e.g. (i) Describe the roots of;

$$a) 3x^2 + 5x + 9 = 0$$

$$\Delta = 5^2 - 4(3)(9)$$

$$= -83 < 0$$

\therefore no real roots

$$b) 2x^2 + 6x - 3 = 0$$

$$\Delta = 6^2 - 4(2)(-3)$$

$$= 60 > 0$$

\therefore two different, real, irrational roots

(ii) Find the values of k which makes;

a) $x^2 + 6x + k = 0$ have equal roots

equal roots occur when $\Delta = 0$

$$i.e. 6^2 - 4k = 0$$

$$36 - 4k = 0$$

$$k = 9$$

b) $x^2 - 4x + 2k = 0$ have unreal roots

unreal roots occur when $\Delta < 0$

$$\text{i.e. } (-4)^2 - 4(2k) < 0$$

$$16 - 8k < 0$$

$$\underline{k > 2}$$

c) $kx^2 + 2x + 4k = 0$ have real roots

real roots occur when $\Delta \geq 0$

$$\text{i.e. } 2^2 - 4(k)(4k) \geq 0$$

$$4 - 16k^2 \geq 0$$

$$k^2 \leq \frac{1}{4}$$

$$\underline{-\frac{1}{2} \leq k \leq \frac{1}{2}}$$

(iii) How many points of intersection does the line $3x - y + 2 = 0$ have with the parabola $y = 2x^2 - 5x - 3$?

$$3x - (2x^2 - 5x - 3) + 2 = 0$$

$$2x^2 - 8x - 5 = 0$$

$$\Delta = (-8)^2 - 4(2)(-5)$$

$$= 104 > 0$$

\therefore there are two points of intersection

as we don't need to find the points of intersection the discriminant can be used to calculate the number

(iv) For what value(s) of a is the line $y = ax$ a tangent to the circle

$$x^2 + y^2 + 20x - 10y + 100 = 0 ?$$

$$x^2 + a^2x^2 + 20x - 10ax + 100 = 0$$

$$(a^2 + 1)x^2 + 10(2 - a)x + 100 = 0$$

line is a tangent when $\Delta = 0$

$$\text{i.e. } 100(2-a)^2 - 4(a^2+1)(100) = 0$$

$$400 - 400a + 100a^2 - 400a^2 - 400 = 0$$

$$3a^2 + 4a = 0$$

$$a(3a + 4) = 0$$

$$a = 0 \quad \text{or} \quad a = -\frac{4}{3}$$

**Exercise 4C; 1ace, 2bdf, 3d, 4c, 5d, 6b, 7, 8c, 9d,
11a, 12a, 13a, 14, 15, 17, 18, 19a ii, 21, 22, 24**