Permutations

Case 2: Ordered Sets of *k* Different Objects, from a Set of *n* Such Objects (*k* < *n*)

(i.e. use some of the objects)

If we have *n* different objects in a line, but only want to arrange *k* of them, the number of ways we could arrange them are;

possibilities for object 1 — possibilities for object 2 — possibilities for object 3 — possibilities for object *k*

$$\text{Number of Arrangements} = n \times (n-1) \times (n-2) \times \cdots \times (n-k+1)$$

$$= n(n-1)(n-2)\cdots(n-k+1) \times \frac{(n-k)(n-k-1)\cdots(3)(2)(1)}{(n-k)(n-k-1)\cdots(3)(2)(1)}$$

$$= \frac{n!}{(n-k)!}$$

$$= {}^{n}P_{k}$$

e.g. (*i*) From the letters of the word **PROBLEMS** how many 5 letter words are possible if;

a) there are no restrictions?

$$\text{Words} = {}^{8}P_{5}$$
$$= 6720$$

b) they must begin with **P**?

the number of ways P can be placed first

$$\text{Words} = 1 \times {}^{7}P_{4}$$
$$= 840$$

Question now becomes how many 4 letter words ROBLEMS

c) **P** is included, but not at the beginning, and **M** is excluded?

the number of positions P can be placed in

Question now becomes how many 4 letter words ROBLES

$$\text{Words} = 4 \times {}^{6}P_{4}$$
$$= 1440$$

(ii) Six people are in a boat with eight seats, for on each side. What is the probability that Bill and Ted are on the left side and Greg is on the right?

$$\text{Ways (no restrictions)} = {}^{8}P_{6}$$
$$= 20160$$

Ways Bill & Ted can go

Ways Greg can go

$$\text{Ways (restrictions)} = {}^{4}P_{2} \times {}^{4}P_{1} \times {}^{5}P_{3}$$
$$= 2880$$

Ways remaining people can go

$$P(\text{B \& T left, G right}) = \frac{2880}{20160}$$
$$= \frac{1}{7}$$

2006 Extension 1 HSC Q3c)

Sophia has five coloured blocks: one red, one blue, one green, one yellow and one white.

She stacks two, three, four or five blocks on top of one another to form a vertical tower.

(*i*) How many different towers are there that she could form that are three blocks high?

$$\text{Towers} = {}^5P_3$$
$$= 60$$

(*ii*) How many different towers can she form in total?

$$\text{2 block Towers} = {}^5P_2 \quad = 20$$
$$\text{3 block Towers} = {}^5P_3 \quad = 60$$
$$\text{4 block Towers} = {}^5P_4 \quad = 120$$
$$\text{5 block Towers} = {}^5P_5 \quad = 120$$
$$\text{Total number of Towers} = 320$$

Exercise 17B; 2, 4, 15, 20, 22

Exercise 17C; 7, 13, 21, 22