# *Quadratic Identities*

Consider the relationship

$$ax^2 + bx + c = f(x)$$

If there are 0, 1 or 2 values that satisfy the relationship, then it is a

**quadratic equation**

*We **solve** equations to find the possible values*

If there 3 or more values that satisfy the relationship, then it is a

**quadratic identity**

*We **prove** identities to show that they are true for all values*

If $a_1x^2 + b_1x + c_1 = a_2x^2 + b_2x + c_2$ for more than 2 values of $x$, then;

$$a_1 = a_2$$
$$b_1 = b_2$$
$$c_1 = c_2$$

e.g. Find $A$, $B$ and $C$ if;

$$\begin{aligned} 2x^2 - 7x - 4 &\equiv A(x+2)^2 + B(x+2) + C \\ &= A(x^2 + 4x + 4) + B(x+2) + C \\ &= Ax^2 + 4Ax + 4A + Bx + 2B + C \\ &= Ax^2 + (4A + B)x + (4A + 2B + C) \end{aligned}$$

the congruence symbol

$\equiv$

is often used with an identity as they are identical expressions

i.e. they are congruent

$A = 2$

$4A + B = -7$

$8 + B = -7$

$B = -15$

$4A + 2B + C = -4$

$8 - 30 + C = -4$

$C = 18$

$\therefore A = 2, B = -15, C = 18$

***OR***

$$2x^2 - 7x - 4 \equiv A(x+2)^2 + B(x+2) + C$$

$x = -2$

$x = -2$ is chosen as it eliminates a variable straight away

$2(-2)^2 - 7(-2) - 4 = C$

$C = 18$

$x = 0$

$x = 0$ is always a simple substitution

$-4 = 4A + 2B + C$

$4A + 2B = -22$

$2A + B = -11$

$x = -1$

now choose a value that will make some variables monic

$2 + 7 - 4 = A + B + C$

$A + B = -13$

$x = -3$ would also be a good choice

$2A + B = -11 \quad (-)$

$A + B = -13$

$A = 2 \quad \therefore B = -15$

$\therefore A = 2, B = -15, C = 18$

***OR***

$$2x^2 - 7x - 4 \equiv A(x+2)^2 + B(x+2) + C$$

equate the coefficients of $x^2$

$$A = 2$$

$x = -2$ eliminates both $A$ & B

$$x = -2$$

$$2(-2)^2 - 7(-2) - 4 = C$$

$$C = 18$$

now choose a value that will make some variables monic

$$x = -1$$

$$2 + 7 - 4 = A + B + C$$

$$B = -15$$

$$\therefore A = 2, B = -15, C = 18$$

**Exercise 4D;**
**3, 5, 6b, 7c, 8a, 9, 11, 12, 13, 15, 17**