Rationalising the Denominator

e.g. (i) $$\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = \underline{2\sqrt{2}}$$

(ii) $$\frac{3}{2\sqrt{5}} = \frac{3}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \underline{\frac{3\sqrt{5}}{10}}$$

(iii) $$\frac{3}{\sqrt{2}-1} = \frac{3}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{3\sqrt{2}+3}{2-1} = \underline{3\sqrt{2}+3}$$

(iv) $$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{4+4\sqrt{3}+3}{4-3} = \underline{7+4\sqrt{3}}$$

(v) Suppose that a, b, c and d are positive integers and c is not a square

a) Given that $\dfrac{a}{b+\sqrt{c}}+\dfrac{d}{\sqrt{c}}$ is rational, prove that $b^2d=c(a+d)$

$$\frac{a}{b+\sqrt{c}}+\frac{d}{\sqrt{c}}=\frac{a\sqrt{c}+d\left(b+\sqrt{c}\right)}{b\sqrt{c}+c}\times\frac{b\sqrt{c}-c}{b\sqrt{c}-c}$$

$$=\frac{abc-ac\sqrt{c}+d\left(b^2\sqrt{c}-bc+bc-c\sqrt{c}\right)}{b^2c-c^2}$$

$$=\frac{(abc)+(b^2d-ac-cd)\sqrt{c}}{b^2c-c^2}$$

which is rational iff $(b^2d-ac-cd)=0$

($\sqrt{c}$ cannot be rational as c is not a perfect square)

$$b^2d-ac-cd=0$$

$$b^2d=ac+cd$$

$$=c(a+d)$$

b) Use part a) to prove that $\frac{a}{1+\sqrt{c}}+\frac{d}{\sqrt{c}}$ is irrational

Assume $\frac{a}{1+\sqrt{c}}+\frac{d}{\sqrt{c}}$ is rational

$$\therefore d = c(a+d) \qquad \text{(from part a))}$$

$$d = ac + cd$$

$$ac = d(1-c)$$

$$a = \frac{d(1-c)}{c}$$

$$\text{now } 1-c \leq 0 \qquad (c \in \mathbb{Z}^+)$$

$$\text{thus } a \leq 0$$

$$\text{however } a > 0 \qquad (a \in \mathbb{Z}^+)$$

$\therefore \frac{a}{1+\sqrt{c}}+\frac{d}{\sqrt{c}}$ is irrational, by contradiction

Exercise 2D;
3dh, 4bh, 5cg,
6bf, 7afk, 8b, 9ac,
11, 12bc,13, 14, 15, 16