

Permutations

Case 3: Ordered Sets of n Objects, Not All Different

2 objects

all different

A B

B A

$$2! = 2$$

2 same

A A

$$1$$

(i.e. some of the objects are the same)

3 objects

all different

A B C

A C B

B A C

B C A

C A B

C B A

$$3! = 6$$

2 same

A A B

A B A

B A A

$$3$$

3 same

A A A

$$1$$

4 objects

all different

A	B	C	D	C	B	A	D
A	B	D	C	C	B	D	A
A	C	B	D	C	A	B	D
A	C	D	B	C	A	D	B
A	D	B	C	C	D	B	A
A	D	C	B	C	D	A	B
B	A	C	D	D	A	C	B
B	A	D	C	D	A	B	C
B	C	A	D	D	C	A	B
B	C	D	A	D	C	B	A
B	D	A	C	D	B	A	C
B	D	A	B	D	B	A	B

$$4! = 24$$

2 same

A	A	B	C
A	A	C	B
A	B	A	C
A	B	C	A
A	C	A	B
A	C	B	A
B	A	A	C
B	A	C	A
B	C	A	A
C	A	A	B
C	A	B	A
C	B	A	A

$$12$$

3 same

A	A	A	B
A	A	B	A
A	B	A	A
B	A	A	A

$$4$$

4 same

A	A	A	A
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$$1$$

If we arrange n objects in a line, of which x are alike, the number of ways we could arrange them are;

$$\text{Number of Arrangements} = \frac{n!}{x!}$$

ways of arranging

n objects

ways of arranging

the like objects

e.g. How many different words can be formed using all of the letters in the word

CONNAUGHTON ?

$$\text{Words} = \frac{11!}{2!3!}$$

$$= \underline{3326400}$$

2! for the two O's

3! for the three N's

2001 Extension 1 HSC Q2c)

The letters *A*, *E*, *I*, *O* and *U* are vowels

(i) How many arrangements of the letters in the word **ALGEBRAIC** are possible?

$$\begin{aligned}\text{Words} &= \frac{9!}{2!} \\ &= \underline{181440}\end{aligned}$$

(ii) How many arrangements of the letters in the word **ALGEBRAIC** are possible if the vowels must occupy the 2nd, 3rd, 5th, and 8th positions?

$$\begin{aligned}\text{Words} &= \frac{4!}{2!} \times 5! \\ &= \underline{1440}\end{aligned}$$

Number of ways of placing the vowels

Number of ways of placing the consonants

Exercise 17D; odd & 18