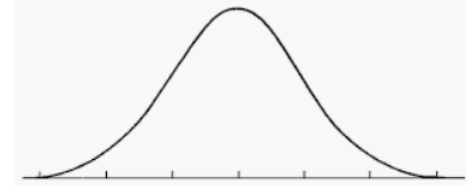


A Bell-Shaped Distribution

When you collect real, continuous data and you represent the results in a histogram, you find that one particular pattern occurs very frequently.



in a symmetrical distribution
mean = median = mode



A typical bell-shaped
probability density function

- The distribution has a modal class somewhere in the middle of the range of values
- The distribution is almost symmetrical

Whilst our data is not symmetrical, like many histograms involving real data, it is almost symmetrical.

- The frequency density tails off fairly rapidly as values move further away from the modal class

Normal Distribution

Any random variable X whose distribution has the probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \forall x \in \mathbb{R}$$

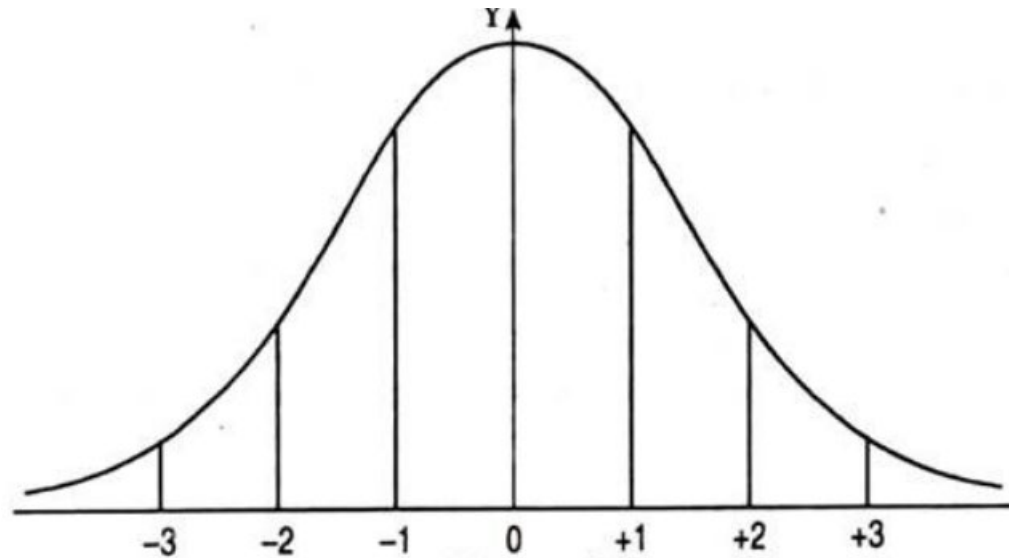
where; μ = mean

σ^2 = variance

is said to have a **normal distribution**, this denoted by

$$X \sim N(\mu, \sigma^2)$$

characteristics of the normal distribution



(I) $f(x) = f(-x)$ (even function)

(II) maximum $f(x) = \mu$ (mode = mean)

(III) $f(x)$ is symmetric about μ

(IV) $f(x)$ has points of inflection at $\mu \pm \sigma$

Standard Normal Distribution

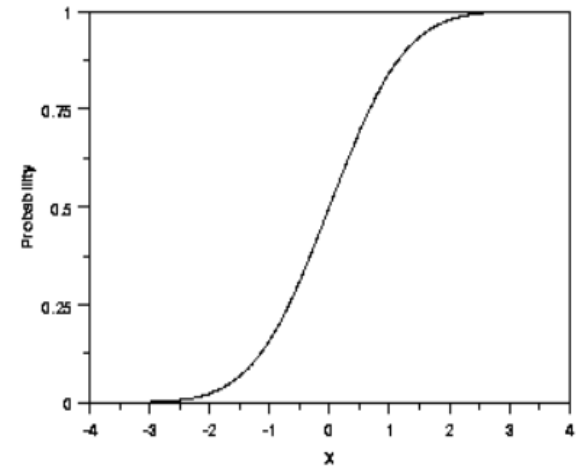
If $\mu = 0$ and $\sigma^2 = 1$, then we have a standard normal random variable $Z \sim N(0,1)$

$$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

calculating probabilities

$$P(Z \leq z) = \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z e^{-\frac{1}{2}z^2} dz$$

$\Phi(z)$ is the cumulative distribution function



a note about integrating $\varphi(z)$

Unfortunately it is not possible to express $\Phi(z)$ in terms of our standard range of functions (algebraic, trigonometric and exponential), using our formal integration methods

Fortunately, we don't need to, as we are interested in the definite integral, i.e. the area under the curve $y = \varphi(z)$, the numerical value of $\Phi(z)$ not the equation of the function.

This can be estimated using the trapezoidal rule

As this is a **standard** normal distribution, these estimates have been placed in a table of values

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517

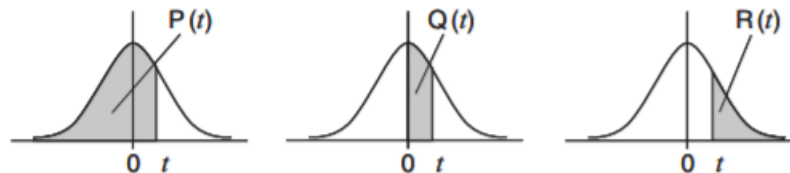
$$\begin{aligned} P(Z \leq 0.25) \\ &= \Phi(0.25) \\ &= \underline{0.5987} \end{aligned}$$

Using the statistical table

1. $P(Z \leq z) = \Phi(z)$
2. $\Phi(-z) = 1 - \Phi(z)$
3. $P(Z \geq z) = \Phi(-z)$
4. $P(z_1 \leq Z \leq z_2) = \Phi(z_2) - \Phi(z_1)$
5. $P(|Z| \leq z) = 2\Phi(z) - 1$

On the fx-100AU PLUS

- MODE-3 , AC (*places the calculator in statistics mode*)
- SHIFT-1-5 (*brings up the standard normal distribution menu*)



- $1 - 0.25$ (*finds $\Phi(0.25) = 0.59871$*)

On the fx-8200AU

- On the HOME screen choose the Distribution menu
- Select **Normal CD**
- Lower: enter -1×10^{99} (*sets the lower limit to negative infinity*)
- Upper: enter 0.25
- μ : enter 0 (*already there by default*)
- σ : enter 1 (*already there by default*)
- Execute (*finds $\Phi(0.25)=0.5987063257$*)

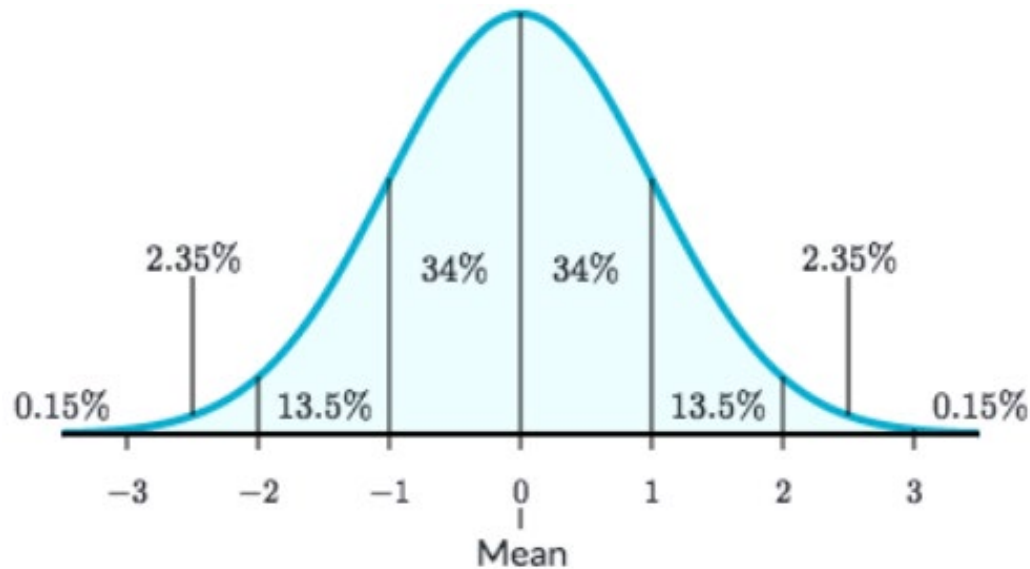
$$\text{e.g. (i) } P(Z \leq 0.44) = \underline{0.67003}$$

$$\begin{aligned} \text{(ii) } P(Z \leq -0.81) &= 1 - \Phi(0.81) && \text{or} = \Phi(-0.81) \\ &= 1 - 0.79103 && = \underline{0.20897} \\ &= \underline{0.20897} \end{aligned}$$

$$\begin{aligned} \text{(iii) } P(-2.34 \leq Z \leq 0.94) &= \Phi(0.94) - \Phi(-2.34) \\ &= 0.82639 - 0.0096419 \\ &= \underline{0.8167481} \end{aligned}$$

$$\begin{aligned} \text{(iv) } P(|Z| \leq 1.15) &= 2\Phi(1.15) - 1 \\ &= 2(0.87493) - 1 \\ &= \underline{0.74986} \end{aligned}$$

Emperical Rule



In a normal distribution;

68% of data lie in the range $\mu \pm \sigma$

95% of data lie in the range $\mu \pm 2\sigma$

99.7% of data lie in the range $\mu \pm 3\sigma$

Exercise 16D; 1ace, 2ace, 3b i, v, 4c i, v, 7c, 8bfj, 9acj, 10ad, 12behl, 13adf, 14ae, 15, 18, 19a, 20