Implicit Differentiation

$$\frac{df}{dx} = \frac{df}{dy} \times \frac{dy}{dx}$$

e.g. (i) $y = x^2$

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^2)$$

$$\frac{d}{dy}(y) \times \frac{dy}{dx} = \frac{d}{dx}(x^2)$$

$$1 \times \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = 2x$$

(ii) $x = y^2$

$$\frac{d}{dx}(x) = \frac{d}{dx}(y^2)$$

$$1 = 2y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2y}$$

$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2) \times \frac{dy}{dx}$$

(iii) $\dfrac{d}{dx}\left(x^2y^3\right)$

$$=\left(x^2\right)\left(3y^2\frac{dy}{dx}\right)+\left(y^3\right)(2x)$$

$$=3x^2y^2\frac{dy}{dx}+2xy^3$$

(iv) Find the equation of the tangent to $x^2 + y^2 = 9$ at the point $\left(1, 2\sqrt{2}\right)$

$$x^2 + y^2 = 9$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

at $\left(1, 2\sqrt{2}\right), \frac{dy}{dx} = -\frac{1}{2\sqrt{2}}$

$\therefore$ required slope $= -\frac{1}{2\sqrt{2}}$

$$y - 2\sqrt{2} = -\frac{1}{2\sqrt{2}}(x - 1)$$

$$2\sqrt{2}y - 8 = -x + 1$$

$$x + 2\sqrt{2}y - 9 = 0$$

"Old Cambridge" **Exercise 7K;**
1acegi, 2bdfh, 3a,
4a, 7, 8